A behavioral cobweb-like commodity market model with heterogeneous speculators

Frank Westerhoff * and Cristian Wieland,

University of Bamberg, Department of Economics, Feldkirchenstraße 21, 96045 Bamberg, Germany

Abstract

According to empirical studies, speculators place significant orders in commodity markets and may cause bubbles and crashes. This paper develops a cobweb-like commodity market model that takes into account the behavior of technical and fundamental speculators. We show that interactions between consumers, producers and heterogeneous speculators may produce price dynamics which mimics the cyclical price motion of actual commodity markets, i.e., irregular switches between bullish and bearish price developments. Moreover, we find that the impact of speculators on price dynamics is non-trivial: depending on the market structure, speculative transactions may either be beneficial or harmful for market stability.

Keywords

Commodity markets; Cobweb models; Heterogeneous Speculators; Bifurcation analysis.

JEL classification

D84; E30; Q11.

* Corresponding author. Email: frank.westerhoff@uni-bamberg.de

1 Introduction

A key characteristic of commodity price dynamics is their strong cyclical behavior. Cashin et al. (2002), who examine the price action of 36 commodities in the period from 1957 to 1999, report that the average price fall across all commodities was 46 percent during slumps, while the average price rise across all commodities was 42 percent during booms. Individual price series are, of course, even more volatile: The price for coconut oil dropped by around 88 percent between June 1984 and August 1986 and the price of coffee arabica increased by around 84 percent from April 1975 to April 1977. Further empirical evidence of commodity price fluctuations is provided by Borenzstein et al. (1994) and Deaton (1999). Alterations between bull and bear markets have important implications for many developing countries dependent on commodity exports. Dramatic price changes may cause severe fluctuations in earnings from commodity exports. A thorough understanding of commodity price dynamics is thus of great significance, especially for policy makers who plan to conduct counter-cyclical stabilization policies (Newberry and Stiglitz 1981).

Several theories have been proposed which give us valuable insight into the dynamics of commodity prices. Our approach is related to cobweb models (e.g. Coase and Fowler 1937, Ezekiel 1938 or Nerlove 1958) which describe the price dynamics in a market of a non-storable good that takes one time unit to produce. As a result, suppliers must form price expectations one period ahead. Such a view is not unrealistic. Consider, for instance, the cultivation of crops. The growing season guarantees a finite lag between the time the production decision is made and the time the crop is ready for sale. The decision about how much should be produced is based on current and past experience. Remember that classical linear cobweb models with naive expectations are able to reproduce oscillatory price movements with decreasing amplitude.

The cobweb approach has been extended in several directions. Exploiting nonlinearities in demand and supply, Chiarella (1988), Day (1994) and Hommes (1994, 1998) analytically show the possibility of chaotic price dynamics for different adaptive expectation schemes of the producers. In Brock and Hommes (1997), the demand and supply curves are linear, but producers switch between different forecasting strategies. Depending on publicly available fitness measures, producers opt either for naive or (costly) rational expectations. The choice is rational in the sense that predictors with a high level of fitness are preferred. The model not only yields complex price dynamics but suggests that irregular dynamics may be part of a fully rational notion of equilibrium.[1] The study of cobweb markets is still an active field of research, see, for instance, the recent contributions by Chiarella and He (2003), Chiarella et al. (2006) and Dieci and Westerhoff (2009).

This paper seeks to offer a new perspective of commodity price fluctuations by adding heterogeneous speculators, i.e. interacting chartists and fundamentalists, to the traditional cobweb framework. In fact, there exists widespread evidence that private and professional speculators apply both technical and fundamental analysis to predict commodity price movements. For instance, Smidt (1965) reports that the majority of the speculators relies at least partially on price charts to render trading decisions in commodity markets. Similar results are reported in questionnaire studies of Draper (1985) and Canoles et al. (1998). In addition, Sanders et al. (2000) find strong evidence of positive feedback trading in several commodity markets and Weiner (2002) detects herding behavior in the petroleum market. Overall, these studies indicate that chart and fundamental speculation is a major factor for price variation in commodity markets.

---

[1] This approach receives, for instance, empirical support from Baak (1999) who estimates the fraction of boundedly rational farmers in the U.S. cattle market.

In line with the early cobweb literature, we construct a behavioral cobweb-like model with a supply response lag. The demand and supply schedules of the consumers and producers are linear and the producers have naive expectations. The market is cleared by the price sensitive demand of the consumers. But the supply available to consumers also depends on the trading decisions of the speculators, i.e. their excess selling (buying) increases (decreases) the supply.[2] The speculators apply both technical and fundamental methods to predict prices. While technical analysis extrapolates past price trends into the future, fundamental analysis assumes that prices converge towards their fundamental values. The speculators are boundedly rational in the sense that they tend to use forecast rules with a high level of fitness. Note that the speculators' switching between technical and fundamental rules introduces a non-linearity into the model.

We are interested in how speculators may influence the evolution of commodity prices.[3] Overall, our model is able to generate price dynamics which mimic the cyclical swings of commodity prices quite well. We derive the following results. Suppose that the cobweb market is stable without speculators. Then a pitchfork bifurcation, followed by a period doubling bifurcation, may emerge as the total number of speculators increases. Further simulation analysis reveals that after many period-doubling bifurcations the dynamics becomes chaotic. For certain parameter values, we observe the emergence of bull and bear markets, as well as irregular price fluctuations between bull and bear markets.

However, if the demand and supply schedules of the consumers and producers violate the stability condition, we show that the presence of a critical mass of speculators may stabilize the market. Instead of a price explosion, the price may settle down on a complicated

[2] Contrary to the classical cobweb model, we thus assume that the commodity is storable. Note that this is indeed the case for most commodities.

[3] The argument that speculators may be destabilizing has a long history, see e.g. Baumol (1957).

attractor, a limit cycle or even a fixed point. This finding is quite remarkable: The common suggestion to crowd out speculators may not always be beneficial to market stability. In fact, complex interactions between technical and fundamental speculators may prevent unstable price trajectories.

The remainder of this paper is organized as follows. Section 2 develops a behavioral cobweb-like commodity market model with heterogeneous boundedly rational speculators. In section 3, we present our analytical results and in section 4, we numerically illustrate the dynamics. The last section offers some conclusions and points out some extensions.

2 A cobweb-like commodity market model with consumers, producers and speculators

2.1 The behavior of consumers and producers

Remember that traditional versions of the cobweb model describe a dynamic price adjustment process on a competitive market for a single non-storable good with a supply response lag. Market clearing occurs in every period

$$D_t = S_t \,, \tag{1}$$

where D and S denote demand and supply, respectively. To keep the model as simple as possible, we focus on linear demand and supply curves. Consumer demand depends negatively upon the current market price P

$$D_t = \frac{a - P_t}{b} \,. \tag{2}$$

The output decision of the producers depends on their price expectations. We assume that producers have naive expectations (i.e. $E[P_t] = P_{t-1}$), which entails a so-called supply response lag. Hence, the supply of the producers in period t is

$$S_t^P = \frac{E[P_t] - c}{d} = \frac{P_{t-1} - c}{d} \,. \tag{3}$$

As usual, we assume that $a, b, d > 0$, $c \geq 0$ and $a/b > c/d$.

In the absence of speculators ($S_t = S_t^P$), the law of motion of the price, obtained by combining (1)-(3), is a one-dimensional linear map

$$P_t = \frac{ad + bc}{d} - \frac{b}{d} P_{t-1}, \tag{4}$$

which has a unique fixed point at

$$F = \frac{ad + bc}{b + d}. \tag{5}$$

We regard the fixed point *F* as the fundamental value of the market. The law of motion may be simplified by rewriting (4) in terms of deviations from the fundamental value. Defining $X_t = P_t - F$, (4) becomes

$$X_t = -\frac{b}{d} X_{t-1}. \tag{6}$$

As is well known, market stability requires

$$b/d < 1. \tag{7}$$

If (7) holds, *P* is attracted by *F*, and *X* converges towards 0.[4] Furthermore, since the parameters *b* and *d* are positive, the price adjustment is oscillatory.

2.2 The behavior of speculators

Our perspective is that producers such as farmers are mainly concerned with the production process. At the commodity exchange, where commodities are usually traded, many additional speculators are active. As revealed by empirical studies, private and professional speculators

---

[4] Note that the parameters *a* and *c* just shift the demand and supply curves vertically upwards or downwards. Hence, the price and its fundamental value both increase in *a* and *c*, yet *X* – the law of motion – is independent of *a* and *c*. Without loss of generality one may assume that *a* and *c* take values such that prices and production quantities are always positive.

use technical and fundamental trading strategies to determine their investment decisions (Smidt 1965, Draper 1985, Canoles et al. 1998, Sanders et al. 2000). Speculators apparently have a marked influence on the evolution of commodity prices.

Interactions between chartists and fundamentalists have already been explored in detail in several stock market models. So-called fundamentalists are agents who believe in mean reversion, i.e. they expect prices to return towards fundamentals. Agents using technical analysis, so-called chartists, bet on the persistence of past price trends. Models by Day and Huang (1990), Huang and Day (1993), de Grauwe et al. (1993), Brock and Hommes (1998), Lux and Marchesi (2000), Chiarella and He (2001) and Chiarella, Dieci and Gardini (2002) demonstrate that the behavior of heterogeneous speculators may endogenously create complex financial market dynamics.

Following this branch of research, we assume that speculators are selling (i.e. increasing the supply) if they expect a decrease in the price and vice versa. The speculators are heterogeneous with respect to their expectation formation. To be precise, they either rely on technical or fundamental prediction rules. Note that we do not keep track of the behavior of individual traders; here we are interested in their aggregated impact on the price dynamics. Furthermore, our approach obviously implies that the commodity is storable. The total supply may then be expressed as

$$S_t = S_t^P + N\,(W_t\, S_t^C + (1-W_t)\, S_t^F\,), \qquad (8)$$

where S^C and S^F are the supply generated by the application of the technical and the fundamental trading rule, respectively. *W* stands for the fraction of agents who follow the technical rule. The market share of fundamentalists is (1-W). *N* denotes the number of speculators, which we normalize to $0 \le N \le 1$.

To characterize the behavior of chartists, we adapt a predictor first suggested by Day and Huang (1990). Chartists optimistically believe in the persistence of a bull market as long as the price is above its fundamental value. Conversely, in a bear market $(P < F)$, chartists pessimistically think that the price will decline further (i.e. $E^C[P_t] = P_{t-1} + e''(P_{t-1} - F)$ with $0 < e'' < 1$). The orders generated by technical analysis are thus given as

$$S_t^C = -e'(E^C[P_t] - P_{t-1}) = -e(P_{t-1} - F), \tag{9}$$

where $e = e'e'' > 0$. Chartists are buying into a rising (bull) market and selling into a falling (bear) market. Note that (9) implies that changes in the demand of chartists are positively correlated with changes in the price.

Fundamentalists expect the price to converge towards its fundamental value. Such regressive expectations may be formalized as $E^F[P_t] = P_{t-1} + f''(F - P_{t-1})$ with $0 < f'' < 1$. The supply due to the fundamental trading rule is

$$S_t^F = -f'(E^F[P_t] - P_{t-1}) = -f(F - P_{t-1}), \tag{10}$$

where $f = f'f'' > 0$. Fundamental analysis suggests selling (buying) if the good is overvalued (undervalued).

The selection of a trading rule depends on market circumstances. The more the price deviates from its fundamental value, the greater the speculators perceive the risk that the price path will collapse (i.e. return to *F*). Hence, the attractiveness of the technical trading rule may be written as

$$A_t^C = g\ Log\,|1/(F - P_{t-1})|, \tag{11}$$

where $g > 0$. Clearly, the attractiveness of technical analysis declines with increasing mispricing. Conversely, the attractiveness of fundamental analysis may be formalized as

$$A_t^F = h\ Log\,|F - P_{t-1}|. \tag{12}$$

Since h is a positive coefficient, speculators regard fundamental analysis as more suitable if the distance between the price and its fundamental value increases. In a broader sense, (11) and (12) capture a forward looking behavior. If market conditions speak against a certain rule, it appears less attractive (i.e. the agents believe that the rule will not perform well in the near future). The idea of forward looking agents has also been formalized by Brock et al. (2006). There, the agents rely on expected future profits as a fitness measure.

The fraction of chartists is given as

$$W_t = \frac{Exp[i\ A_t^C]}{Exp[i\ A_t^C] + Exp[i\ A_t^F]} . \quad (13)$$

According to (13), the fraction of speculators who apply the technical trading rule increases if the attractiveness of that rule increases. The fraction of speculators who follow the fundamental trading rules is defined as $(1-W)$. The coefficient $i \geq 0$ measures how sensitive the mass of traders is to selecting the most attractive rule. For instance, if $i = 0$, then the traders do not discriminate between the options ($W = 0.5$). The higher i, the more speculators select the rule with the highest level of attractiveness. Inserting (11) and (12) into (13) yields

$$W_t = \frac{1}{1 + |F - P_{t-1}|^j} , \quad (14)$$

where $j = i(g + h)$.

The modified law of motion of the price, in terms of deviations from the fundamental value, becomes

$$X_t = -\frac{b}{d} X_{t-1} + \frac{b\ N\ X_{t-1}(e - f\ |X_{t-1}|^j)}{1 + |X_{t-1}|^j} . \quad (15)$$

Due to its second term, (15) is a one-dimensional non-linear difference equation.

3 Some analytical results: A local analysis for the case *j*=2

In a number of papers (e.g. de Grauwe et al. 1993, Hommes 2001 or Westerhoff 2003), a bell-shaped switching function such as (14) with *j*=2 is used.[5] Here we focus on this special, yet quite prominent case. We are now able to derive a first theoretical insight into the dynamics of our cobweb-like model which hopefully will improve our understanding of the workings of commodity markets. In section 4, we also explore the dynamics of the model for $j \neq 2$.

For convenience, we express (15) as

$$X_t = H(X_{t-1}, N), \tag{16}$$

where $H(X,N) = X\left(-\frac{b}{d} + \frac{b\,N\,(e - f\,X^2)}{1 + X^2}\right)$. Since $H(-X,N) = -H(X,N)$, the map is symmetric around the origin. The consequences of this will become clear in the following.

The parameter *N* plays a key role in our stability and bifurcation analysis. Denote $N_1 = \frac{b-d}{bde}$, $N_2 = \frac{b+d}{bde}$ and $N_3 = \frac{-be + f(b+2d) + \sqrt{(e+f)(eb^2 + f(b+2d)^2)}}{2efbd}$, where $N_1 < N_2 < N_3$. We are now able to derive the following results (proven in the appendix).

Proposition 1: The difference equation (16) possesses three fixed points at $X_1 = 0$ and $X_{2,3} = \pm\frac{\sqrt{b + d - bdeN}}{\sqrt{-b - d - bdfN}}$, where $X_{2,3}$ only exist if $N > N_2$.

Proposition 2: If *b*<*d* then the difference equation (16) possesses

(a) a locally stable fixed point at $X_1 = 0$ for $0 < N \leq N_2$,

(b) a pitchfork bifurcation at $X_1 = 0$ for $N = N_2$,

---

[5] In the previous section we have presented a simple way how one may derive such a function from a discrete choice approach. We hope that this is interesting and may turn out useful for future research.

(c) two symmetric locally stable fixed points at $X_{2,3} = \pm \frac{\sqrt{b + d - bdeN}}{\sqrt{-b - d - bdfN}}$ for $N_2 < N \le N_3$,

(d) a period doubling bifurcation at $X_{2,3} = \pm \frac{\sqrt{b + d - bdeN}}{\sqrt{-b - d - bdfN}}$ for $N = N_3$.

Proposition 3: If $b>d$ then the difference equation (16) possesses

(a) a locally unstable fixed point at $X_1 = 0$ for $0 < N < N_1$,

(b) a (subcritical) period doubling bifurcation at $X_1 = 0$ for $N = N_1$,

(c) a locally stable fixed point at $X_1 = 0$ for $N_1 < N \le N_2$,

(d) for $N \ge N_2$ see Proposition 2 (b), (c) and (d).

Observe the fundamental difference between Propositions 2 and 3. If the basic cobweb market is locally stable ($b/d<1$), an increase in the number of speculators tends to decrease the efficiency of the market in the sense that a locally stable fixed point equal to the fundamental value may be transformed into one of the two locally stable fixed points unequal to the fundamental value and then into one of the two period-two cycles. From that point of view it might appear desirable to ban speculators from the market, for instance by imposing a transaction tax. But such popular policy advice may easily turn out to be a mixed blessing. If the market becomes unstable ($b/d>1$), either due to a technology shock or a shift in the preferences of the consumers, the existence of a critical mass of speculators ($N>N_1$) guarantees at least some kind of stability. Instead of an exploding price trajectory, the price either converges towards its fundamental value, to one of the two non-fundamental fixed points, to a limit cycle in the bull or the bear market, or displays complex motion. Thus it might be better to tolerate the activity of speculators and to accept some mispricing to prevent unstable price paths.[6]

4 Some further numerical results

Let us carry on the analysis by numerical investigation. Figure 1 presents the bifurcation diagrams for the model's six parameters. The bifurcation parameter is increased in 400 steps as indicated on the axis while the remaining parameters are given as

$N = 0.57$, $b = 1$, $d = 2$, $e = 10$, $f = 1$ and $j = 2$.

For each parameter combination, 100 observations are plotted. A transient period of 500 periods has been erased to allow the system to settle down on its attractor.

Figure 1 goes about here

The first panel of figure 1 continues the bifurcation analysis of the previous section. As can be seen, the more speculators enter the market, the more complex the dynamics. After many period doubling bifurcations the dynamics becomes chaotic. Comparable routes into chaos are observed for parameters *b* and *e*. However, there also exist different routes into chaos, indicating that the model has the potential to produce quite complex dynamics for a large variety of parameter combinations. Put differently, the emergence of endogenous price movements is not a special case. From a qualitative point of view one may say that an increase in both *b* and *e* seems to increase the range in which the fluctuations takes place, whereas an increase in *f* tends to shrink the upper and lower price boundaries. Parameters *d*

[6] Recall that for $b>d$ there is a subcritical period doubling bifurcation at $N=N_1$, i.e. an unstable period two cycle emerges around the now stable fixed point $X_1=0$. Numerical evidence suggests that this period two cycle marks the basin of attraction for the fixed point (see Agliari et al. 2006 for a related case in a two-dimensional system). Orbits starting from within this basin will converge towards the fixed point. Moreover, using the parameter setting of figure 3, this basin increases with *N*. Hence, more speculators imply a larger basin of attraction (and thus may contribute also in this sense to market stability).

and *j* hardly allow such conclusions, which again reveals the model's strength to generate complex dynamics.

Figure 2 compares actual prices of agricultural products with simulated prices. The first, second and third panels display monthly hog prices, corn prices and hog-corn price ratios between 1970 and 2002, respectively. All three time series reveal cyclical motion, as is often reported for commodity markets. Especially the hog-corn price ratio data switches between high and low price periods.[7] A simulated time series with 250 observations is depicted in the bottom panel (we use the same parameter setting as in the bifurcation analysis). Our simple model is able to mimic such cyclical price patterns. Although the model is completely deterministic, prices move up and down erratically.

Figure 2 goes about here

Let us try to understand what is going on in the (artificial) market. Since we have set $b/d<1$, the price would – in the absence of speculators – settle down at its fundamental value. However, the presence of speculators destabilizes the market. If the price is close to its fundamental value, chartism is popular and thus the price is driven away from its fundamental value. But as the mispricing increases, fundamental trading becomes more fashionable and a convergence sets in. Since this again favors chartism, the pattern continually repeats itself, but in a very intricate manner. If the price crosses the fundamental value, a temporary bull market turns into a temporary bear market or vice versa. To sum up, the price seems to move up and down erratically in a bull or bear market, and also seems to swing erratically between bull and bear markets. Such a trajectory qualitatively resembles those displayed in the top three panels.

[7] Since corn is the primary ingredient of the hog's diet (it constitutes around 60-65 percent of the total costs of pig production), the hog-corn price ratio is a general indicator of profitability and, therefore, future changes in pork production. Therefore, it is not surprising that Ezekiel´s (1938) hog-corn price ratio figure, ranging from 1900 to 1935, looks, despite technological progress, quite similar to the one displayed here.

Let us finally turn to the case in which the basic cobweb market is unstable ($b/d>1$). If there are no speculators ($N=0$), the price diverges from its fundamental value. To be precise, the dynamics evolves as follows. Suppose the price is above its fundamental value in period t–1. In the next period, the consumers face an increased supply of the commodity and consequently the price drops below its fundamental value. As a result, the producers reduce their output in period t+1 and the price increases again. Due to $b/d>1$, the price is now higher than in period t–1. This pattern repeats itself and the price path explodes.

Contrary to the intuition, proposition 3 shows that speculators may stabilize an otherwise unstable market although their behavior is in general destabilizing. Figure 3 illustrates this puzzling feature. It presents a simulation run for the following parameter setting

$N = 0.242$, $a = 20$, $b = 2.2$, $c = 10$, $d = 2$, $e = 10$, $f = 1$ and $j = 2$.

The first, second, third and fourth panel of figure 3 display the price of the commodity, the demand of the consumers, the output of the producers and the net supply of the speculators, respectively. Note that the price of the commodity switches erratically between bull and bear markets (as in figure 2), yet does not explode.[8] The reason for this becomes obvious in the bottom three panels. Note that when the price is high (i.e. above its fundamental value), the output of the producers is also high. However, this does not automatically lead to a crash in the next period since the net supply available to the consumers now also depends on the activity of the speculators. If the price is high, the speculators buy the commodity (see bottom panel). This reduces the net supply and thus hinders the system from crashing. The same is true in the opposite case. If the price is low (i.e. below its fundamental value), the output of the producers is also low. But now the speculators are selling the commodity so that the price

[8] Such kind of dynamics may also survive in higher dimensional systems, see, e.g. Tramontana et al. (2009).

remains in the bull market. Overall, we observe intricate dynamics, mainly due to the behavior of the speculators, but the system does not run away from its fundamental value. In this sense, speculators stabilize the dynamics. The popular recommendation to crowd out speculators may thus – in general – not be beneficial to market stability.

Figure 3 goes about here

5 Conclusions

For more than 100 years, the regularly recurring cycles in the production and prices of particular commodities have been studied with great interest (for an early review see Ezekiel 1938). The cobweb framework has become the basic workhorse to explore this phenomenon. While the early theoretical contributions have been linear in design, a number of promising non-linear models have also been formulated (Chiarella 1988, Day 1994, Hommes 1994, Brock and Hommes 1997). Without question, all these papers help to explain the empirical evidence.

But to our understanding one important aspect has been overlooked. As indicated in many empirical studies, speculators have a marked impact on the price formation process (Smidt 1965, Draper 1985, Canoles et al. 1998, Sanders et al. 2000, Weiner 2002). Our paper extends the basic linear cobweb model with naive expectations and a supply response lag by the incorporation of heterogeneous interacting speculators. Even in its simple form the model has the potential to generate cyclical, yet complex, price dynamics. The dynamics live from the fact that a sufficient fraction of the speculators applies destabilizing technical trading rules. However, as we have shown analytically, banning speculators from the market may only be a mixed blessing. Although the speculators might, on average, be regarded as destabilizing there also exist situations in which they stabilize the dynamics.

The simplicity of our model has been achieved by ignoring some of the details of commodity markets. We would thus like to point out a number of extensions. First, in our model, producers and speculators operate on the same time scale. A more reasonable perspective would be that the producers update their production decision on, say, a weekly or monthly basis while speculators trade on a daily basis (or even more frequently). An interesting modification of this model would be to combine different time scales for speculators and producers. Second, actual commodity transactions are accomplished by the co-existence of spot and futures markets. Here, our attention is restricted to the spot market only. Brock et al. (2009) provide some first potential frameworks on how to combine the two markets. The analysis gets much more complicated, but also more realistic. Third, numerical evidence (not reported) reveals that the average long-term position of speculators, compared to an average speculative transaction, may be relatively low.[9] However, it may be worthwhile to explore a scenario in which the speculators actively control their inventory position.[10] This will most likely increase the dimension of the system and may open up the door for periodic or quasiperiodic motion via a Neimark-Sacker bifurcation. Fourth, in many agricultural markets we often see interventions by governments that try to influence prices. Cobweb models are suitable to pre-study the consequences of such actions. However, if one omits the impact of speculators, the strategies may be ill designed. Fifth, our model is deterministic. By

[9] At least for situations in which there are erratic switches between bull and bear market situations (as e.g. in figures 2 and 3). The reason seems to be that the price dynamics is symmetric with respect to the fundamental value. The case is, of course, different if the system settles down on one of the two non-fundamental steady states. The positions would then increase over time. However, one may argue that we discuss here the deterministic skeleton of a more general noisy model and that noise may push the system from one to the other attractor now and then.

[10] However, this “problem” is also evident in many stock market models (see section 2.2 for references) in which market makers mediate the transactions of speculators. The inventory of the market makers may become extreme over time. This pressing issue has largely been ignored in the literature so far.

adding dynamic noise one may try to calibrate the model even more closely to the stylized facts. For instance, one may investigate the consequences of demand and supply shocks. Finally, since the structure of the current model is quite simple it may be possible to test the model statistically using real data. For some results in this direction see Reitz and Westerhoff (2007).

Appendix

Proof for Proposition 1:

Solving

$$-\frac{b}{d}X + \frac{bNX(e - fX^2)}{1 + X^2} - X = 0 \qquad \text{(A1)}$$

with respect to *X* one obtains $X_1 = 0$ and

$$X_{2,3} = \pm \frac{\sqrt{b + d - bdeN}}{\sqrt{-b - d - bdfN}}. \qquad \text{(A2)}$$

The latter two fixed points only exist if $N > N_2$ (see below).

Proof for Proposition 2:

(a) Recall that $\partial H / \partial X$ denotes the eigenvalue of our one-dimensional map. Since $b<d$,

$$\left.\frac{\partial H}{\partial X}\right|_{X=0, N=0} = -\frac{b}{d} > -1 \qquad \text{(A3)}$$

and

$$\left.\frac{\partial H}{\partial X}\right|_{X=0, N=N_2} = 1. \qquad \text{(A4)}$$

Thus, the fixed point at $X=0$ is locally stable for $0 < N \le N_2$.

(b) From (A4) we know that $X=0$ for $N=N_2$ is a nonhyperbolic fixed point with eigenvalue 1. A few transformations reveal that

$$\left.\frac{\partial H}{\partial N}\right|_{X=0, N=N_2} = 0,$$

$$\left.\frac{\partial^2 H}{\partial X^2}\right|_{X=0, N=N_2} = 0,$$

$$\left.\frac{\partial^2 H}{\partial X \partial N}\right|_{X=0, N=N_2} = eb \neq 0,$$

$$\left.\frac{\partial^3 H}{\partial X^3}\right|_{X=0, N=N_2} = \frac{6(e+f)(-b-d)}{ed} \neq 0.$$

Thus, $X=0$ undergoes a pitchfork bifurcation at $N = N_2$. See, for instance, Wiggins (1990) for the requirements of a pitchfork bifurcation.

(c) Note that

$$\left.\frac{\partial H}{\partial X}\right|_{X=X_{2,3},N=N_3} = -1 \qquad \text{(A5)}$$

Thus, the fixed points at $X_{2,3} = \pm\frac{\sqrt{b+d-bdeN}}{\sqrt{-b-d-bdfN}}$ are locally stable for $N_2 < N \leq N_3$.

(d) From (A5) we know that $X_{2,3} = \pm\frac{\sqrt{b+d-bdeN}}{\sqrt{-b-d-bdfN}}$ for $N = N_3$ are nonhyperbolic fixed points with eigenvalue -1. A few transformations show that

$$\left.\frac{\partial H^2}{\partial N}\right|_{X=X_{2,3},N=N_3} = 0,$$

$$\left.\frac{\partial^2 H^2}{\partial X^2}\right|_{X=X_{2,3},N=N_3} = 0,$$

$$\left.\frac{\partial^2 H^2}{\partial X \partial N}\right|_{X=X_{2,3},N=N_3} = \frac{2b((e+f)(eb^2+f(b+2d)^2)+(be-f(b+2d))\sqrt{(e+f)(eb^2+f(b+2d)^2)}}{(e+f)(b+d)^2} \neq 0,$$

$$\left.\frac{\partial^3 H^2}{\partial X^3}\right|_{X=X_{2,3},N=N_3} = \frac{-48(eb^2+f(b+d)^2)\sqrt{(e+f)(eb^2+f(b+2d)^2)}}{e(e+f)d(b+d)^2}$$

$$+\frac{48(-b^3e^2-f^2(b+d)^2)(b+2d)-efb(2b^2+4bd+3d^2)}{e(e+f)d(b+d)^2} \neq 0.$$

Thus, the fixed points at $X_{2,3} = \pm\frac{\sqrt{b+d-bdeN}}{\sqrt{-b-d-bdfN}}$ undergo a period-doubling bifurcation for $N = N_3$. See again Wiggins (1990) for the requirements of a period doubling bifurcation.

Proof for Proposition 3:

(a) Since $b>d$,

$$\left.\frac{\partial H}{\partial X}\right|_{X=0,N=0} = -\frac{b}{d} < -1$$

and

$$\left.\frac{\partial H}{\partial X}\right|_{X=0,N=N_1} = -1. \qquad \text{(A6)}$$

Thus, $X_1 = 0$ is an unstable fixed point for $0 < N < N_1$.

(b) From (A6) we know that $X=0$ for $N=N_1$ is a nonhyperbolic fixed point with eigenvalue -1. A few transformations reveal that

$$\left.\frac{\partial H^2}{\partial N}\right|_{X=0,N=N_1} = 0,$$

$$\left.\frac{\partial^2 H^2}{\partial X^2}\right|_{X=0,N=N_1} = 0,$$

$$\left.\frac{\partial^2 H^2}{\partial X \partial N}\right|_{X=0,N=N_1} = -2be \neq 0,$$

$$\left.\frac{\partial^3 H^2}{\partial X^3}\right|_{X=0,N=N_1} = \frac{12(b-d)(e+f)}{de} \neq 0.$$

Thus, X_1=0 undergoes a (subcritical) period doubling bifurcation at $N = N_1$.

(c) $X_1 = 0$ is a stable fixed point for $N_1 \leq N \leq N_2$ because of (A6) and (A4).

(d) See proofs of Proposition 2 (b), (c) and (d).

References

Agliari, A., Chiarella, C., and Gardini, L. (2006): A re-evaluation of adaptive expectations in light of global nonlinear dynamic analysis. Journal of Economic Behavior and Organization, 60, 526-552.

Baak, S. (1999): Tests for bounded rationality with a linear dynamic model distorted by heterogeneous expectations. Journal of Economic Dynamics and Control, 23, 1517-1543.

Baumol, W. (1957): Speculation, profitability and stability. Review of Economics and Statistics, 39, 263-271.

Borenzstein, E., Kann, M., Reinhart, C. and Wickham, P. (1994): The behavior of non-oil commodity prices. IMF Occasional Paper 112, IMF: Washington.

Brock, W. and Hommes, C. (1997): A rational route to randomness. Econometrica, 65, 1059-1095.

Brock, W. and Hommes, C. (1998): Heterogeneous beliefs and routes to chaos in a simple asset pricing model. Journal of Economic Dynamics Control, 22, 1235-1274.

Brock, W., Hommes, C. and Wagener, F. (2009): More hedging instruments may destabilize markets. Journal of Economic Dynamics and Control, 33, 1912-1928.

Brock, W., Dindo, P. and Hommes, C. (2006): Adaptive rational equilibrium with forward looking agents. International Journal of Economic Theory, 2, 241-278.

Canoles, B., Thompson, S., Irwin, S., and France, V. (1998): An analysis of the profiles and motivations of habitual commodity speculators. Journal of Futures Markets, 18, 765-801.

Cashin, P., McDermott, J. and Scott, A. (2002): Booms and slumps in world commodity prices. Journal of Development Economics, 69, 277-296.

Chiarella, C. (1988): The cobweb model: Its instability and the onset of chaos. Economic Modelling, 5, 377-384.

Chiarella, C, Dieci, R, and Gardini, L. (2002): Speculative behaviour and complex asset price dynamics: A global analysis. Journal of Economic Behavior and Organization, 49, 173-197.

Chiarella, C. and He, X.-Z. (2001): Asset price and wealth dynamics under heterogeneous expectations. Quantitative Finance, 1, 509-526.

Chiarella, C. and He, X.-Z. (2003): Dynamic beliefs and learning under the aL-processes: The heterogeneous case. Journal of Economic Dynamics and Control, 27, 503-532.

Chiarella, C., He, X.-Z., Hung, H. and Zhu, P. (2006): An analysis of the cobweb model with boundedly rational heterogeneous producers. Journal of Economic Behavior and Organization, 61, 750-768.

Coase, R. and Fowler, R. (1937): The pig cycle in Great Britain: An explanation. Economica 4, 55-82.

Day, R. (1994): Complex economic dynamics: An introduction to dynamical systems and market mechanisms. MIT Press: Cambridge.

Day, R. and Huang, W. (1990): Bulls, bears and market sheep. Journal of Economic Behavior and Organization, 14, 299-329.

Deaton, A. (1999): Commodity prices and growth in Africa. Journal of Economic Perspectives, 13, 23-40.

De Grauwe, P., Dewachter, H. and Embrechts, M. (1993): Exchange rate theory: Chaotic models of foreign exchange markets. Blackwell: Oxford.

Dieci, R. and Westerhoff, F. (2009): Stability analysis of a cobweb model with market interactions. Applied Mathematics and Computation, 215, 2011-2023.

Draper, D. (1985): The small public trader in futures markets. In: Peck, A. (Ed.): Futures Markets: Regulatory Issues. American Enterprise Institute for Public Policy Research: Washington, 211-269.

Ezekiel, M. (1938): The cobweb theorem. Quarterly Journal of Economics, 52, 255-280.

Hommes, C. (1994): Dynamics of the cobweb model with adaptive expectations and non-linear supply and demand. Journal of Economic Behavior and Organization, 24, 315-335.

Hommes, C. (1998): On the consistency of backward-looking expectations: The case of the cobweb. Journal of Economic Behavior and Organization, 33, 333-362.

Hommes, C. (2001): Financial markets as nonlinear adaptive evolutionary systems. Quantitative Finance, 1, 149-167.

Huang, W. and Day, R. (1993): Chaotically switching bear and bull markets: The derivation of stock price distributions from behavioral rules. In: Day, R. and Chen, P. (Eds.): Non-linear dynamics and evolutionary economics. Oxford University Press: Oxford, 169-182.

Lux, T. and Marchesi, M. (2000): Volatility clustering in financial markets: A micro-simulation of interacting agents. International Journal of Theoretical and Applied Finance, 3, 675-702.

Nerlove, M. (1958): Adaptive expectations and cobweb phenomena. Quarterly Journal of Economics, 227-240.

Newbery, D. and Stiglitz, J. (1981): The theory of commodity price stabilization: a study in the economics of risk. Claredon Press: Oxford.

Reitz, S. and Westerhoff, F. (2007): Commodity price cycles and heterogeneous speculators: A STAR-GARCH model. Empirical Economics, 33, 231-244.

Sanders, D., Irwin, S. and Leuthold, R. (2000): Noise trader sentiment in futures markets. In: Goss, B. (Ed.) Models of futures markets. Routledge: London, 86-116.

Smidt, S. (1965): Amateur speculators: A survey of trading strategies, information sources and patterns of entry and exit from commodity futures markets by non-professional speculators. Cornell Studies in Policy and Administration, Cornell University.

Tramontana, F., Gardini, L., Dieci, R. and Westerhoff, F. (2009): The emergence of “bull and bear” dynamics in a nonlinear model of interacting markets. Discrete Dynamics in Nature and Society, 2009, Article ID 310471.

Weiner, R. (2002): Sheep in wolves’ clothing? Speculators and price volatility in petroleum futures. Quarterly Review of Economics and Finance, 42, 391-400.

Westerhoff, F. (2003): Anchoring and psychological barriers in foreign exchange markets. Journal of Behavioral Finance, 4, 65-70

Wiggins, S. (1990): Introduction to applied nonlinear dynamical systems and chaos. Springer: New York.

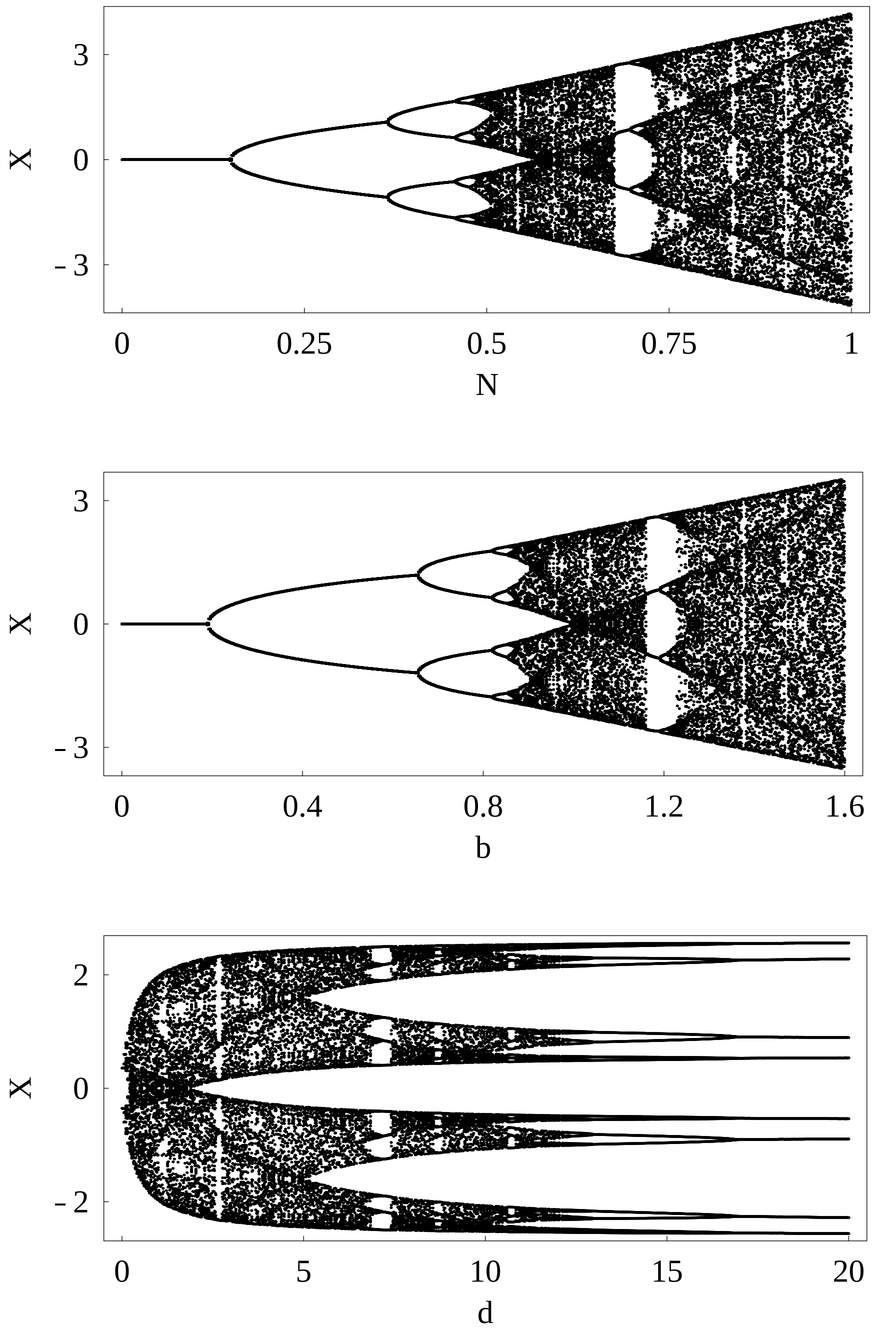

Figure 1: Bifurcation diagrams. The bifurcation parameter is increased in 400 steps. For each value, 100 observations are plotted (a transient period of 500 periods has been erased). Parameter setting as in section 4 and as indicated on the axis.

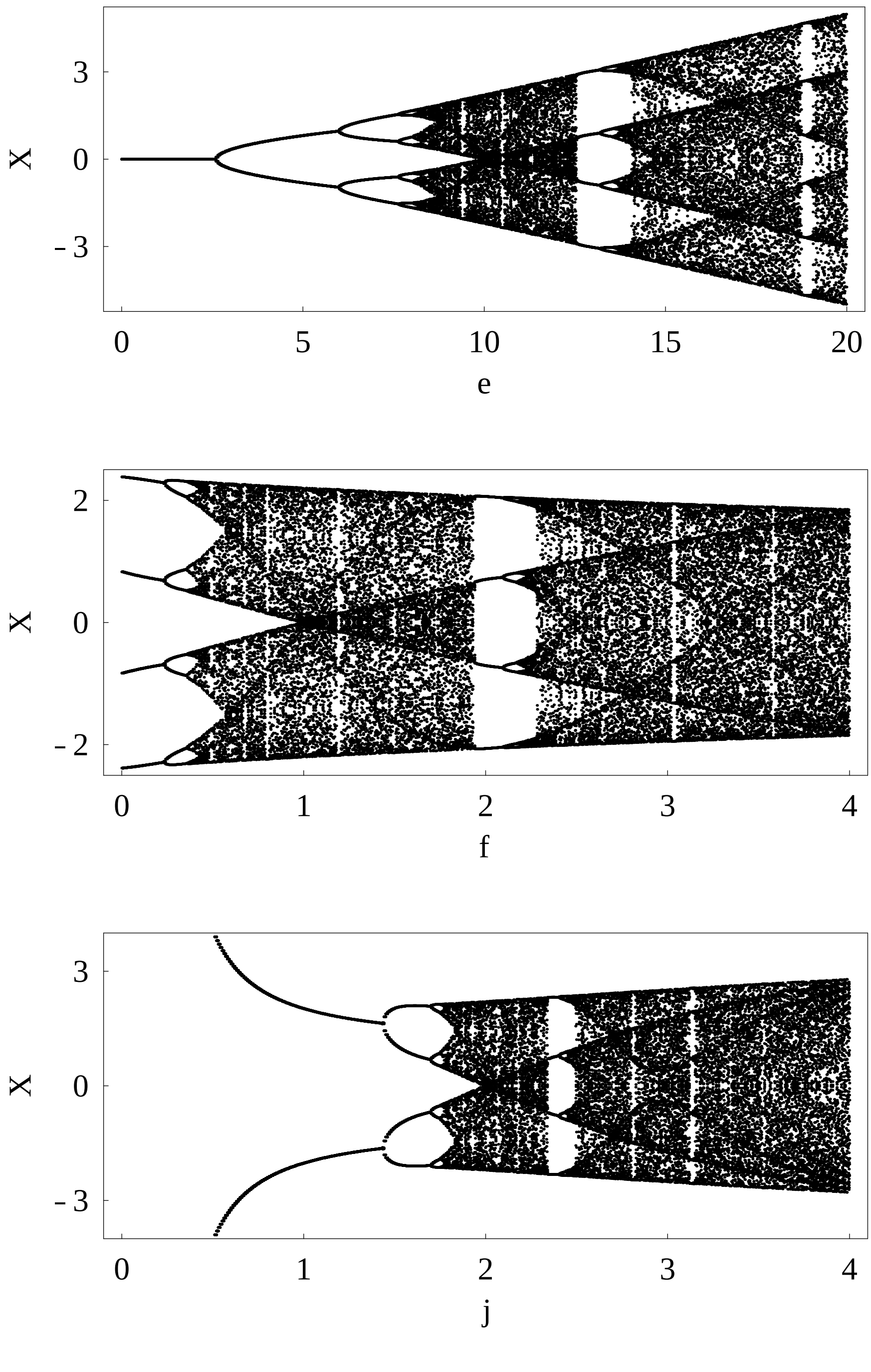

Figure 1: continued.

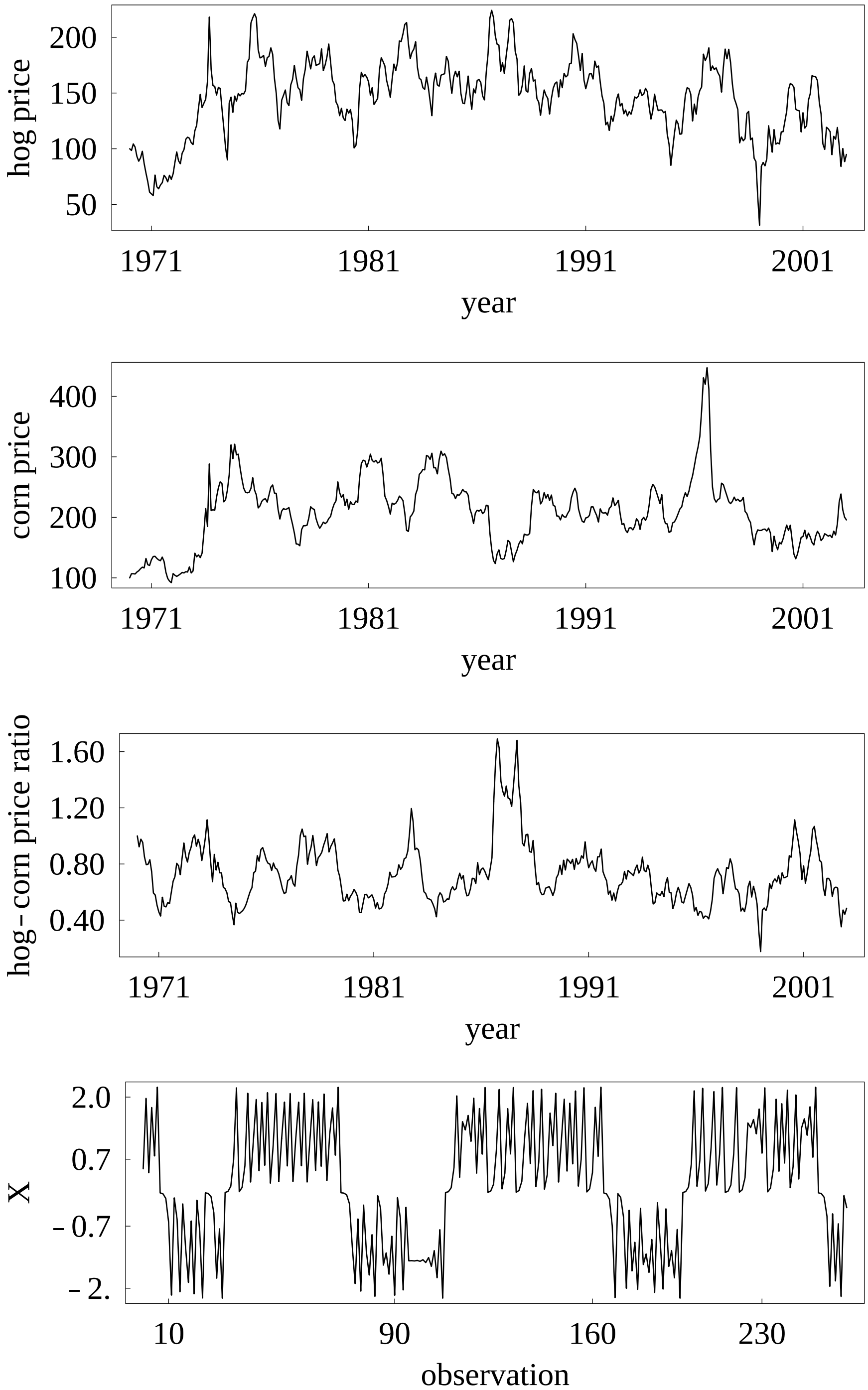

Figure 2: Cycles in cobweb markets. The first three panels show the hog price, the corn price and the hog-corn price ratio between 1970 and 2002 (monthly data, 1970=100). The fourth panel displays simulated prices for 250 observations.

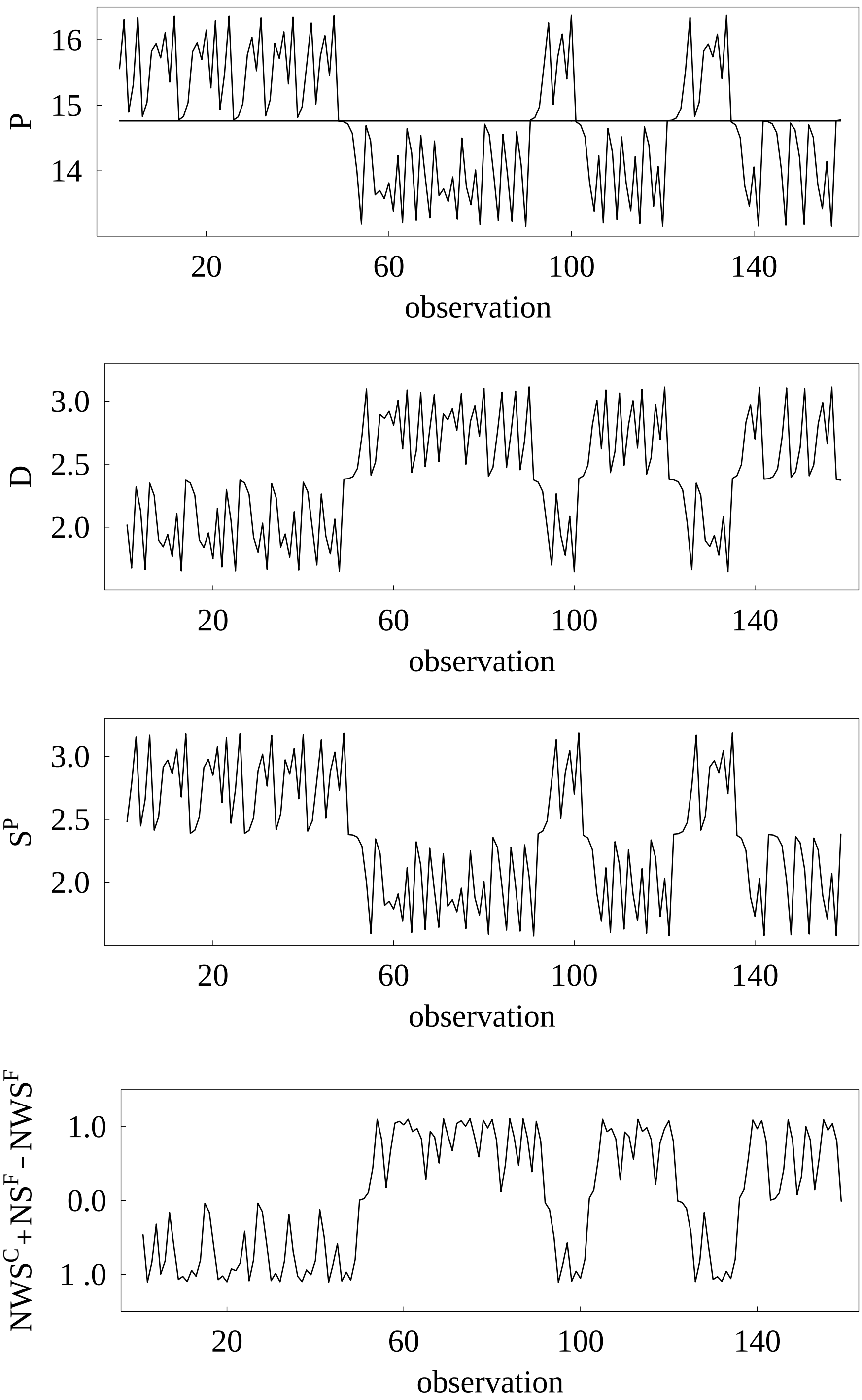

Figure 3: The impact of speculators when *b*>*d*. The first, second, third and fourth panel shows the price of the commodity, the demand of the consumers, the output of the producers and the supply of the speculators, respectively.