Controlling tax evasion fluctuations*

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Abstract

We incorporate the behaviour of tax evasion into the standard two-dimensional Ising model and augment it by providing policy-makers with the opportunity to curb tax evasion via an appropriate enforcement mechanism. We discuss different network structures in which tax evasion may vary greatly over time if no measures of control are taken. Furthermore, we show that even minimal enforcement levels may help to alleviate this problem substantially.

[Keywords: Opinion dynamics, Sociophysics, Ising model.]

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1 Introduction

The aim of this study is to contribute to the, in our opinion, lack of multi-agent-based studies on tax evasion. Descriptions of some of these studies may be found in Bloomquist (2006). We incorporate the behaviour of tax evasion into the standard two-dimensional square lattice Ising spin model. We aim to extend the study of Zaklan et al. (2008), which illustrates that in a world where agents are conditionally cooperative, enforcement may be an appropriate mechanism to establish a higher level of overall tax compliance.[1] We embed our tax evasion model into different network structures and find for these networks that fluctuation in aggregate tax evasion behaviour may arise if no enforcement is used. Furthermore, we provide evidence, by simulation, that even minimal enforcement may help to reduce the presence of fluctuations in tax evasion. Such fluctuations can be completely prevented in the considered networks by setting the enforcement measures to sufficiently high, but realistic, levels. Most of society then becomes compliant over time. For our simulations we make use of the "tunnelling" process at temperatures slightly below the critical temperature. This has been studied for two decades in the field of physics, and also by Hohnisch et al. (2005) for the IFO-Index.

Evidence that agents usually behave conditionally cooperative may be found in Gächter (2006). Presenting the findings of public goods experiments, he argues that conditional cooperation primarily motivates people to either contribute to the provision of a public good or to free-ride. Conditional cooperators, as in our model, are more likely to evade taxes if they have the impression that many others evade. On the other hand if most others behave honestly, an individual is less likely to cheat on her taxes. Frey and Torgler (2006) provide empirical evidence on the relevance of conditional cooperation for tax morale. They find a positive correlation between people's tax morale, which is measured by asking whether tax evasion is justified if the chance arises, and their perception regarding how many others evade paying tax. Conditional cooperation from the viewpoint of the standard economic theory may be explained by changes in risk aversion due to changes in equity (Falkinger,

[1] An individual's decision regarding tax evasion is shaped by the surroundings in which the individual lives, as will be explained.

1995).

To relate our simulations to real-world figures, it seems interesting to look briefly at the degree of actual non-compliance and enforcement levels first: the overall rate of tax non-compliance in the USA with regard to individual income tax is estimated at approximately 17 percent for both the years 1988 (Andreoni et al., 1998) and 2001 (Slemrod, 2007). The more visible the type of income is, the higher the degree of tax compliance usually is. For instance, the compliance rate with respect to wage and salary income is estimated to be over 98%. By contrast, the level of compliance regarding self-employed income, which is not subject to information reporting, is estimated at about 78% (Lederman, 2003). Over time, in the USA, the proportion of all individual returns that are audited has fallen substantially. In 1965, 4.75% of all individual returns were audited. This figure reached a level of 1.03% in 2007, which is the highest rate since 1998 (IRS, 2007). In 2000 the audit rate for individuals was only about 0.5 percent. Also, the rich are audited more often (for millionaires, the audit rate was about 10% in 2007) than individuals with incomes below $100,000 (audit rate of about 1%). Although penalties for detected tax evaders may be quite severe, punishments are seldom imposed. In 1995, for example, only 4.1 percent of all US taxpayers who were found evading, were penalised for fraud.

Tax evasion can vary greatly across nations, reaching very high proportions in some developing countries. Compliance and enforcement data, if they exist, are difficult to obtain and only few countries outside the USA have been studied. It is therefore, for instance, instructive to estimate the non-compliance rate for the value-added tax or to look at the extent of the shadow economy in different countries in order to derive figures of compliance for income tax evasion. According to the calculations of Schneider (2007), for example, in 2004/2005 Greece, Italy and Spain had the largest shadow economies among the 21 OECD countries (26.3%, 23.2% and 20.5%, respectively). At the lower end are the USA, Switzerland and Japan (7.9%, 8.5% and 8.8%, respectively), while the figures of Ireland, Germany and Canada correspond more or less to the OECD average (14.1%, 15.3% and 14.1%, respectively).

The remainder of our manuscript is organised as follows: in section 2 we present our model, which is based on the standard two-dimensional Ising model on a square lattice. In section 3 we decribe the evolution of the aggregate tax evasion behaviour that our model generates under different enforcement regimes. In section 4 we additionally embed our model into the Barabási-Albert network and the Voronoi-Delaunay random lattice and discuss the resulting tax evasion dynamics.

2 The model

We use the standard Glauber kinetics of the Ising model on a 20×20 square lattice. In every time period each lattice site is occupied by an individual (=spin S_i) who can either be an honest tax payer $S_i = +1$ or a tax evader $S_i = -1$. The small number of agents may be imagined to represent the elite of a country, whose tax evasion behaviour it may be interesting to look at, given the different enforcement regimes of the tax authority. The tax morale of managers and rich people in general is currently of great concern in Germany. For our analysis we assume that everybody is honest initially. Each period individuals have the opportunity to become the opposite type of agent as they were in the previous period. Each agent's social network, which is made up of four next neighbours, may either prefer tax evasion or reject it.

Tax evaders have the greatest influence to turn honest citizens into tax evaders if they constitute a majority in the given neighbourhood. If the majority evades, one is likely to also evade. On the other hand, if most people in the vicinity are honest, the respective individual is likely to become a decent citizen if she was a tax evader before.[2] For very low temperatures, the autonomous part of decision-making almost completely disappears.[3] Individuals then base their decisions solely on what most of their neighbours do. A rising temperature has the opposite effect. Individuals then decide more autonomously. For $T > T_c$ (≈ 2.269), half of the people are honest and the other half cheat.

As an enforcement measure, we introduce a probability of an efficient audit (p).

[2]How strong the influence from the neighbourhood is can be controlled by adjusting the temperature, T. Total energy is given by the Hamiltonian $H = -\sum_{<i,j>} J_{ij} S_i S_j - B \sum_i S_i$.

[3]The autonomous part of individual decision-making is responsible for the emergence of the tax evasion problem, because some initially honest tax payers decide to evade taxes and then exert influence on others to do so as well.

If tax evasion is detected, the individual must remain honest for a certain number of periods. We denote the period of time for which detected tax evaders are punished by the variable k. One time unit is one sweep through the entire lattice. Already Föllmer (1974) applied the Ising model to economics.

3 Dynamics of the model

— Figure 1 goes about here —

The top-left panel of Figure 1 illustrates the baseline setting, i.e. no use of enforcement, for the square lattice. We depict the dynamics of tax evasion over $50,000$ time steps. Although everybody is honest initially, it is not possible to predict which level of tax compliance will be reached at some time step in the future. Agents are usually either mostly compliant or mostly non-compliant, whereas the system typically remains in either state for a while. Switching from a mostly compliant to a mostly non-compliant society, or vice versa, is favoured by both the small number of agents and the temperature, which needs to be somewhere close to the critical level (we use $T \approx T_c$). If, by chance, more than 50% of agents start to prefer the opposite action of the currently dominating one, this strategy will then start to prevail for a while. As soon as there is a majority for the previously dominating strategy regarding tax evasion, aggregate behaviour is then likely to reverse again. If more agents or a temperature further below the critical level are picked, it would take longer for a switch in aggregate evasion behaviour to occur. Apparently, a suitable measure of control is needed to prevent agents from repeatedly falling into non-compliance.

— Figure 2 goes about here —

Figure 2 illustrates different simulation settings for the square lattice, where for each considered combination of degree of punishment ($k = 1$, 10 and 50) and audit rate ($p = 0.5$, 10 and 90%) the corresponding dynamics of tax evasion is depicted over $50,000$ time steps. Surprisingly, even very small levels of enforcement (e.g. $p = 0.5\%$

and $k = 1$) suffice to almost completely prevent fluctuation in aggregate tax evasion behaviour and to establish mainly compliance. Only seldomly tax evasion then becomes the predominant aggregate choice of action. Both, a rise in audit probability (greater p) and a higher penalty (greater k), work to flatten the time series of tax evasion and to shift the band of possible non-compliance values towards more compliance. If the audit rate is increased to the level of 1%, even for very small penalties ($k = 1$) then an upsurge in tax evasion will not occur any longer (not displayed). Since high income earners are audited more often ($p \approx 10\%$) than average income tax payers ($p \approx 1\%$), we look at how results change if higher levels of enforcement are used ($p = 10\%$). Interestingly, higher audit rates only reduce the level of tax evasion marginally. The simulations illustrate that even extreme enforcement measures (e.g. $p = 90\%$ and $k = 50$) cannot fully resolve the problem of tax evasion.

Our results suggest the importance of the fact that expenditure on enforcement measures may be variable in nature. Since the period of honesty may be long, enforcement may then not be necessary. On the other hand, as soon as non-compliance starts to spread, low levels of enforcement may be desirable, because drastically increasing levels of enforcement only marginally help to reduce tax evasion, but are very costly.

4 Modifications

To examine whether the results generated by the square lattice are robust, we extend our analysis to other frequently used network structures. Specifically, we make use of the Voronoi-Delaunay random lattice and the Barabási-Albert network model. The construction of the Voronoi-Delaunay lattice (i.e. tessellation of the plane for a given set of points) is defined as follows (Lima et al., 2000). For each point, one first needs to determine the polygonal cell, consisting of the region of space nearer to that point than to any other point. Whenever two such cells share an edge, they are considered to be neighbours. From the Voronoi tessellation, one can obtain the dual lattice by the following procedure: when two cells are neighbours, a link is placed between the two points located in the cells. From the links, one obtains the triangulation of space. The network constructed in this manner, which we use for simulation, is called the Voronoi-Delaunay lattice.

The Barabási-Albert network (Barabási and Albert, 1999) is grown such that the probability of a new site to be connected to one of the already existing sites is proportional to the number of connections the existing site has already accumulated over time: individuals with many friends are more likely to gain new friends than loners.

In these variations of our simple square lattice model we also choose 400 agents and depict the resulting tax evasion dynamics over $50,000$ time steps.

The remaining pictures in Figure 1 illustrate the dynamics in the baseline setting in these additional network structures of our model. Both networks, the Voronoi-Delaunay lattice and the Barabási-Albert network, support our findings in the case of the square lattice, namely that fluctuations in tax evasion behaviour may occur if no enforcement mechanism is implemented.

— Figure 3 goes about here —

Figure 3 illustrates the tax evasion dynamics for the Voroi-Delaunay lattice (first column) and the Barabási-Albert (other two columns) network, if different degrees of enforcement are used.

For the Voronoi-Delaunay random lattice we also find that fluctuations in tax evasion can be reduced substantially by implementing very low probabilities of an audit. For an audit rate of $p = 1\%$ no fluctuations occur any longer and society remains mainly honest over time. Obviously higher punishments, i.e. higher levels of k, also lower the amount of non-compliance.

The next two columns illustrate that enforcement in the Barabási-Albert network is less efficient than in the Voronoi-Delaunay network or in the simple square lattice. As can be seen, fluctuations still occur for $p = 1\%$, even for high levels of punishment (e.g. $k = 10$). The last column illustrates the tax evasion dynamics, holding the audit rate constant at $p = 4.5\%$. This is the minimal audit rate which (almost always) prevents tax evasion from fluctuating, even for the lowest considered level of punishment (i.e. $k = 1$), and is much higher than in the other two network models, yet still at a realistic level.

5 Conclusion

Tax evasion can vary widely across nations, reaching extremely high values in some developing countries. Wintrobe and Gërxhani (2004) explain the observed higher level of tax evasion in generally less developed countries with a lesser amount of trust that people accord to governmental institutions. Empirical evidence for the importance of trust for tax compliance may, for example, be found in Hyun (2005) and in Torgler (2004). So far we have neglected the effects of public opinion, which may vary greatly across countries, on tax evasion. It therefore seems worthwhile to extend the setting of our simple model in a further study, to specifically analyse the aspect of varying tax compliance across countries.

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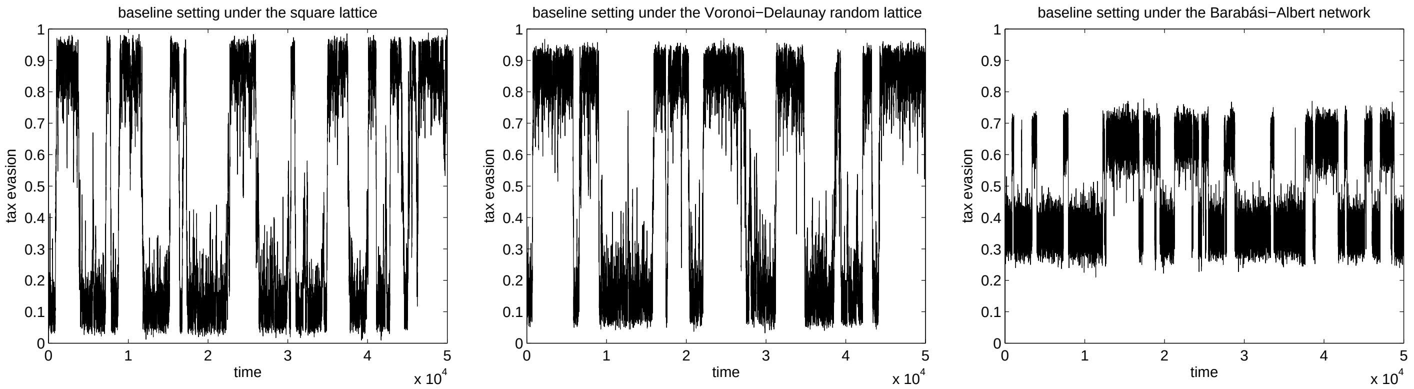

Figure 1: The baseline setting is given if no enforcement measure is implemented to control tax evasion. The figures above illustrate the baseline settings for the different network structures we use. For the square lattice we take $T = 2.265$, for the Voronoi-Delaunay lattice $T_c = 3.802$ and for the Barabási-Albert network $0.8 \cdot T_c = 0.8 \cdot m \cdot \log(NSITES)/2$ (where $m = 4$ and $NSITES = 400$). All simulations are performed over $50,000$ time steps.

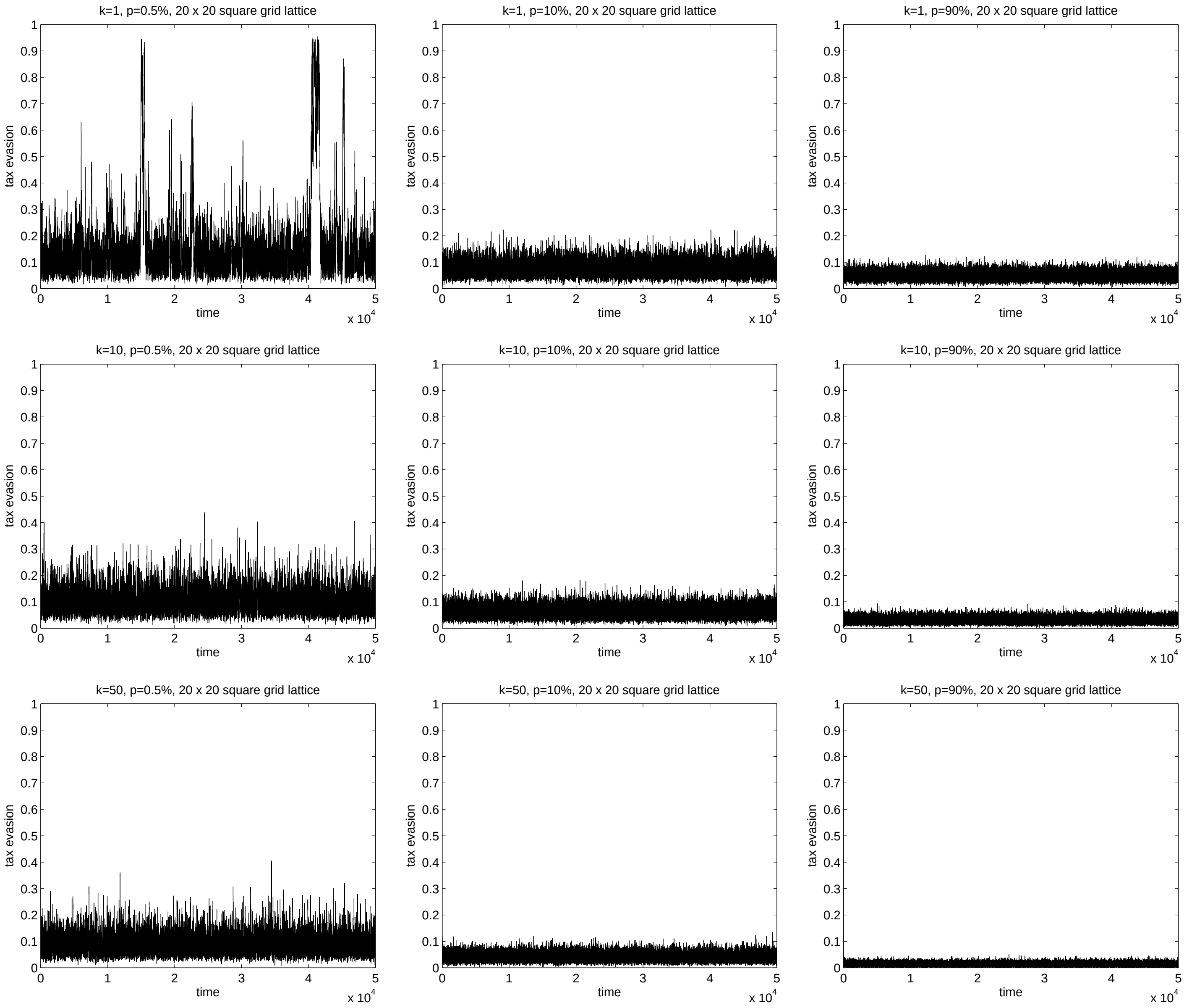

Figure 2: The square lattice model of tax evasion with various degrees of enforcement. ($T = 2.265$ and $50,000$ time steps)

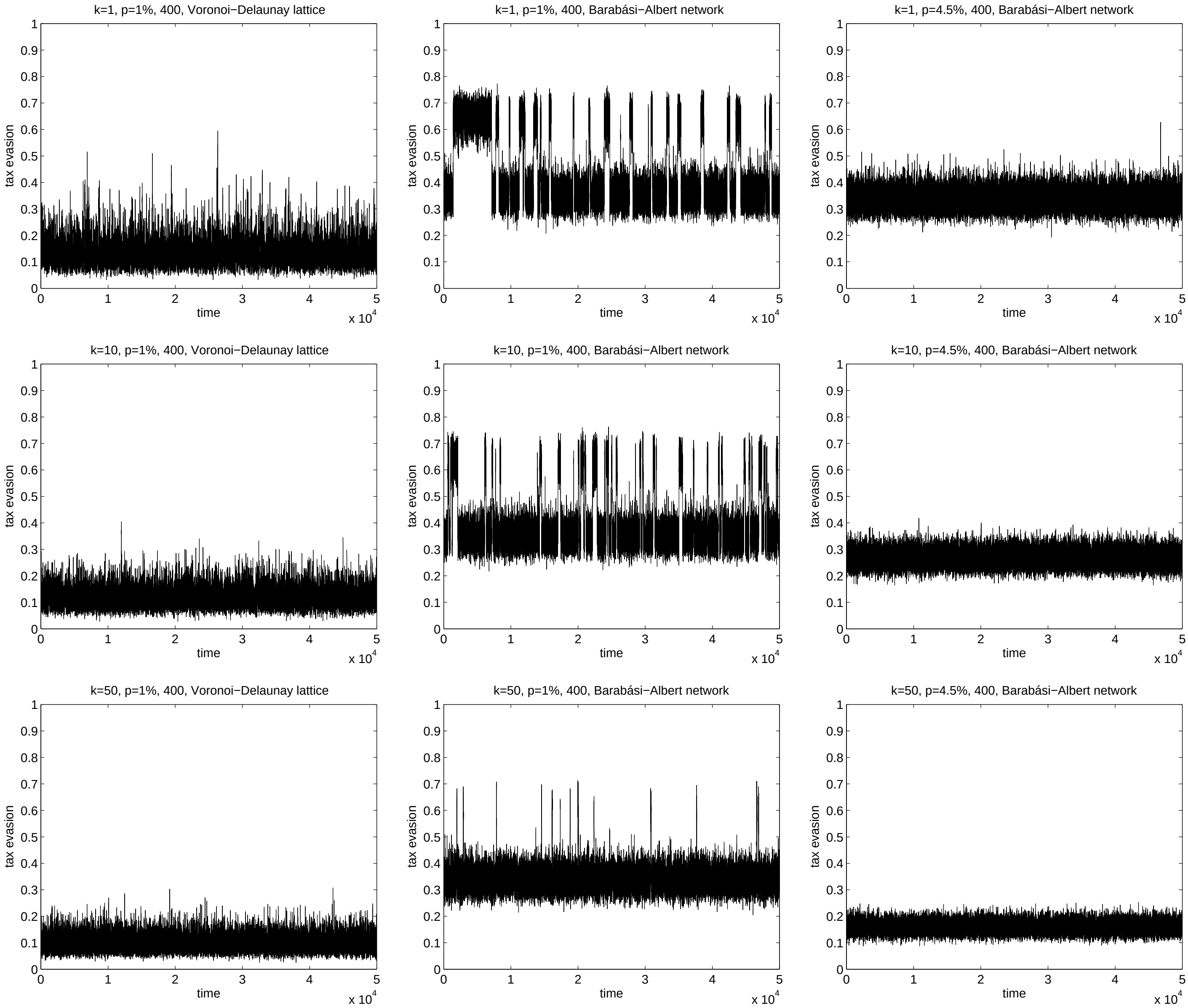

Figure 3: The first column illustrates the resulting tax evasion dynamics for different enforcement regimes if the Voronoi-Delaunay network is used. The next two columns depict the tax evasion dynamics in the case of the Barabási-Albert network. Again, we use $50,000$ time steps.