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# Business cycles, heuristic expectation formation and contracyclical policies*

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## Abstract

We develop a simple Keynesian-type business cycle model in which agents use simple heuristics to predict national income. To be precise, the agents either form (destabilizing) extrapolative expectations or (stabilizing) regressive expectations, a decision which depends on the rules forecasting performance in the recent past. As it turns out, an unending evolutionary competition between the rules may generate endogenous complex business cycles. We also explore the effectiveness of some common governmental intervention strategies. Our model suggests that policy makers may be able to stabilize output fluctuations, yet due to system immanent nonlinearities this may prove to be quite difficult.

## Keywords

agent-based computational economics, business cycles,

heuristic expectation formation, contracyclical policies, complex dynamics

## JEL classification

E32, E12, D84

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## 1 Introduction

Short-run output variations are a world-wide phenomenon, basically all modern industrial economies regularly experience significant swings in economic activity. Alternating periods of expansion and contraction are naturally welfare-decreasing. Every recession in which workers become involuntarily unemployed is associated with an income loss that cannot be regained. Booms and recessions may have an exogenous trigger. However, complex output movements may also arise endogenously due to nonlinearities (for excellent surveys see, e.g., Gandolfo 1985, Puu 1989, Medio 1992, Day and Chen 1993, Day 1999 or Rosser 2000). Especially in the latter case, policy makers may want to stabilize the economy. But as demonstrated by Baumol (1961), the variability of national income may increase if intervention mechanisms are ill-designed. A thorough understanding of the causes of business cycles thus seems to be quite important.

In this paper, we develop a business cycle model to analyze the role of expectation formation for output fluctuations and to explore the usefulness of common contracyclical intervention rules. Our starting point is that people are boundedly rational in the sense of Simon (1955). Although agents lack the knowledge and computational capabilities to determine optimal actions, they strive to do the right thing. As argued by Kahneman, Slovic and Tversky (1986), people rely on a limited number of heuristic principles which reduce the complex tasks of assessing probabilities and predicting values to simpler judgmental operations. These heuristics face a natural selection pressure, i.e. only well-performing rules survive. With respect to expectation formation, two popular types of heuristics exist. Extrapolative expectations simply assume a continuation of the current trend and are thus likely to be destabilizing. Regressive expectations are often regarded as a stabilizing force since they add a negative feedback to the dynamics. Note that regressive expectations are relatively sophisticated and thus may be more costly.

Guided by these empirical regularities, we develop a business cycle model in which agents form simple (cheap) extrapolative or sophisticated (costly) regressive expectations to predict their income. Moreover, the agents do not stick to a certain rule but compare their relative fitness and tend to select the better one. Clearly, agents prefer heuristics which have produced low squared prediction errors in the recent past. To make matters as simple as possible, we use a Keynesian multiplier framework to describe the rest of the economy. A central finding is that movements in national income may arise endogenously due to a permanent competition between different heuristics. In periods in which extrapolative expectations are dominating, the economy is unstable and strong cyclical output movements occur. However, extrapolative predictors perform poorly at turning points. When regressive expectations gain in popularity, a period of convergence sets in. Since prediction errors of extrapolative expectations become low when output is relatively stable, at least some agents return to this type of forecasting rule and thus output oscillations may increase again.

Policy makers frequently conduct contracyclical interventions to smooth business cycles. But at least within our setup, this proves to be rather difficult. Both trend-adjusting and level-adjusting strategies have, in principle, the potential to reduce the amplitude of business fluctuations. However, success and failure of the interventions depend critically on how the rules are executed. Already tiny mistakes in the strength of the interventions may make a huge difference, turning a stable environment into an unstable one.

We continue as follows. In section 2, we first develop a business cycle model in which agents follow heterogeneous expectation formation rules. In section 3, we discuss the workings of the model. In section 4, we study the effectiveness of a few common contracyclical intervention policies. In section 5, we explore several routes to business cycles. The last section concludes the paper and points out some interesting avenues for future research.

## 2 A simple business cycle model

In order to explore the impact of heuristic expectation formation on business cycles, the economy is modeled as simply as possible. For this reason, we express national income $Y$ at time step $t$ as

$$Y_t = I_t + C_t, \tag{1}$$

where

$$I_t = \bar{I} \tag{2}$$

comprises all autonomous expenditures.[1] Consumption is a function of national income. But instead of postulating that consumption depends on the last period's output (according to the so-called Robertson lag), we assume that the agents consume a constant fraction $a$ of their current expected income $E[Y_t]$. Hence, we have

$$C_t = aE[Y_t]. \tag{3}$$

The consumption parameter $a$ is restricted to $0 < a < 1$.

Note that for $\bar{Y} = Y_t = E[Y_t]$, (1)-(3) imply that

$$\bar{Y} = \frac{\bar{I}}{1-a}, \tag{4}$$

which corresponds to the well-known Keynesian multiplier solution. As in Baumol (1961), we interpret the steady state $\bar{Y}$ as the near full employment output level.

As mentioned in the introduction, people frequently rely on simple heuristics such as extrapolative and regressive forecasting rules to predict economic variables. The average market expectation with respect to national income may therefore be defined as

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[1] Recent extensions of Samuelson's (1939) multiplier-accelerator model include, for instance, Sushko, Puu and Gardini (2003), Puu, Gardini and Sushko (2005) and Westerhoff (2005). Incorporating an investment function makes, of course, the analysis more realistic, yet also more intricate. In order to highlight the role of expectation formation for the emergence of output fluctuations, we treat investment as exogenously given and constant.

$$E[Y_t] = W_t^e E^e[Y_t] + W_t^r E^r[Y_t]\,. \tag{5}$$

The relative weight of extrapolative expectations $E^e[Y_t]$ is denoted by $W_t^e$ and the relative weight of regressive expectations $E^r[Y_t]$ is represented by $W_t^r$ with $W_t^e + W_t^r = 1$.

When the agents compute their expected income for period *t*, they possess information up to period *t*-1. Extrapolative expectations may then be formalized as

$$E^e[Y_t] = Y_{t-1} + b(Y_{t-1} - Y_{t-2})\,, \tag{6}$$

where $b > 0$ indicates how strongly the agents extrapolate past output changes into the future. Regressive expectations may be written as

$$E^r[Y_t] = Y_{t-1} + c(\overline{Y} - Y_{t-1})\,. \tag{7}$$

Accordingly, the agents expect the gap between the near full employment output level $\overline{Y}$ and the observed output level $Y_{t-1}$ to be reduced by a factor $0 < c < 1$.

Now we turn to the crucial part of the model. The agents do not stick to a certain predictor but compare their relative performance. Here, we measure the fitness of the rules by their squared prediction error. The agents consequently favor rules with low prediction errors. To be precise, the attractiveness of extrapolative expectations is given as

$$A_t^e = -(E^e[Y_{t.-1}] - Y_{t-1})^2\,. \tag{8}$$

While it is straightforward to extrapolate trends, computing regressive expectations is a more sophisticated task. For instance, people have to estimate the near full employment output level $\overline{Y}$. For being able to do this, they first have to develop some general knowledge about how the economy may work. The attractiveness of regressive expectations is thus modeled as

$$A_t^r = -d - (E^r[Y_{t.-1}] - Y_{t-1})^2\,, \tag{9}$$

where $d \geq 0$ captures the costs associated with applying the more sophisticated predictor.[2]

The relevance of the different forecasting rules depends on their relative fitness: The higher the fitness of a predictor, the more agents will follow it.[3] As in the discrete-choice approach of Manski and McFadden (1981), the weights of the two predictors are given as

$$W_t^e = \frac{\exp[hA_t^e]}{\exp[hA_t^e] + \exp[hA_t^r]} \tag{10}$$

and

$$W_t^r = \frac{\exp[hA_t^r]}{\exp[hA_t^e] + \exp[hA_t^r]}. \tag{11}$$

The parameter $h \geq 0$ may be regarded as the intensity of choice and measures how sensitive the mass of agents is to selecting the most attractive predictor. Note that an increase in $h$ may be interpreted as an increase in the rationality of the agents. For $h = 0$, the agents do not discriminate between the predictors and thus they split evenly between them. But if $h$ goes to infinity, all agents select the predictor with the highest fitness.

Combining (1)-(11) reveals that

$$Y_t = f(Y_{t-1}, Y_{t-2}, Y_{t-3}), \tag{12}$$

i.e. the recurrence relation which determines national income is a three-dimensional nonlinear difference equation.[4] Due to the functional form of (12), we continue with numerical analysis. One advantage of our simple setup is that it should be quite easy to replicate the simulations.

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[2] In section 5, we will see that this assumption is not crucial, complex dynamics may also arise for $d = 0$. However, assuming that the formation of regressive expectations is more costly than the formation of extrapolative expectations seems to be more natural and thus we take this aspect into account.

[3] The modeling of predictor choice is inspired by the work of Brock and Hommes (1997, 1998). While the former contribution is concerned with the expectation formation of heterogeneous producers within a cobweb framework, the latter focuses on financial market dynamics. An excellent introduction into agent-based computational models is Judge and Tesfatsion (2006).

[4] It is easy to calculate that the unique steady state of (12) is given by the near full employment output level $\overline{Y}$.

## 3 The emergence of business cycles: An example

We assume the following basic parameter setting for our simulations analysis:

$a = 0.9$, $b = 1.4$, $c = 0.15$, $d = 1.2$, $h = 1$.

In addition, autonomous expenditures are fixed at $\bar{I} = 1000$. Since the "multiplier" is 10, the near full employment output level is $\overline{Y} = 10000$. Moreover the initial conditions of our three-dimensional system are given as $Y_1 = 10000$, $Y_2 = 10000$ and $Y_3 = 10001$ (i.e. we slightly disturb the steady state in period $t = 3$).[5] Note that if national income is for at least two consecutive periods equal to $\overline{Y}$, then the agents expect output to be stable and thus it is. But due to the heuristic expectation formation process, output is – at least within our setup – not necessarily stable.

Figure 1 shows a typical simulation run. The top panel presents the evolution of output for 500 observations (after omitting a longer transient period). Although the dynamics is entirely deterministic, we observe the emergence of expansions, followed by recessions. The sequence of booms and slumps is recurrent, but not periodic. Both the duration of business cycles and their amplitude vary. In a stylized way, the simulated output dynamics resembles actual fluctuations in economic activity (Stock and Watson 1999).

**= = = = = Figure 1 goes about here = = = = =**

What drives the dynamics? The bottom panel depicts the weight of the extrapolative predictor. Note that in most cases either all agents form regressive or extrapolative expectations. Suppose that all agents use the regressive predictor, i.e. $W_t^e = 0$ and $W_t^r = 1$. Then the law of motion of national income is given as

$$Y_t = \bar{I} + ac\overline{Y} + (a - ac)Y_{t-1}, \tag{13}$$

[5] The dynamics discussed in this section does not depend on our choice of initial conditions. Section 5, however, demonstrates that initial conditions may become critical for other parameter combinations.

which is a one-dimensional linear difference equation. The steady state of (13) is equal to $\bar{Y} = \bar{I}/(1-a)$. Moreover, monotonic convergence towards the steady state sets in if

$$0 < (a - ac) < 1. \tag{14}$$

Since $0 < a < 1$ and $0 < c < 1$, (14) is always true. Hence, if all agents use the regressive forecasting rule, national income monotonically converges towards its steady state level.

Let us now turn to the case in which all agents prefer the extrapolative heuristic, i.e. $W_t^e = 1$ and $W_t^r = 0$. Then we obtain the following second-dimensional linear difference equation

$$Y_t = \bar{I} + (a + ab)Y_{t-1} - abY_{t-2}. \tag{15}$$

Obviously, the steady state is given as $\bar{Y} = \bar{I}/(1-a)$. Rewriting Schur's stability conditions[6] delivers

$$a < 1 \tag{16}$$

and

$$a < 1/b. \tag{17}$$

Furthermore, we can compute that (15) generates cycles if

$$a < 4b/(1+b)^2. \tag{18}$$

Since $a = 0.9$ and $b = 1.4$, our model produces unstable cyclical motion when all agents rely on extrapolative expectations.

But as is visible in figure 1, output neither settles down on its fixed point nor does it explode. Instead, we observe frequent, yet irregular regime shifts. For some time, regressive expectations may appear superior. Then the economy behaves stably and output approaches

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[6] Remember that a second-dimensional linear difference equation $X_t + a_1 X_{t-1} + a_2 X_{t-2} = Y$ is stable if (1) $1 + a_1 + a_2 > 0$, (2) $1 - a_1 + a_2 > 0$ and (3) $1 - a_2 > 0$. The conditions for cycles is $a_1^2 < 4a_2$.

its near full employment level. But when output is close to $\bar{Y}$, the prediction errors of both types of predictors become small. Since regressive expectation formation is relatively costly, the extrapolative predictor appears to be more attractive. As a result, agents now trend-extrapolate output changes and increasing output oscillations emerge. At some point in time, however, extrapolative expectations produce quite strong prediction errors (especially at turning points, i.e. local maxima and minima). Although regressive expectation formation is costly, it eventually becomes advantageously again. Then a period of convergence sets in until the pattern repeats itself, yet in a complex way. Clearly, it is the endogenous adaptation process of the agents that creates business cycles.

## 4 Some policy experiments

So far we have seen that the agent's expectation formation process may lead to an ebb and flow of economic activity. Note that if the dynamics is endogenous, then policy makers may have a chance to stabilize economic activity. Following Baumol (1961), we discuss two common intervention measures. The first policy aims at adjusting income levels. Suppose the government always decides to compensate for the difference between actual output and its near full employment level. Such a level-adjusting policy may be formalized as

$$G_t^L = g^L(\bar{Y} - Y_{t-1}), \tag{19}$$

where $g^L > 0$ is the policy maker's control parameter. As long as output is below $\bar{Y}$, the government increases its spending (and vice versa).[7] The aggregated intervention volume

$$D = \sum_{t=1}^{T} G_t^L, \tag{20}$$

[7] The national income equation (1) now has on its right-hand side autonomous expenditures, consumption and government interventions. The system remains, of course, three dimensional.

is simply given as the sum over all past interventions.[8]

Figure 2 displays a simulation run in which the government applies (19) with $g^L = 0.05$. All other things remain as in the previous figure. The outcome is striking: Output fluctuations are now more moderate. The main reason may be summarized as follows. By pushing output back towards $\bar{Y}$, mean reversion sets in more early than before. Remember that extrapolative expectations perform poorly at turning points. Since agents switch faster to regressive expectations when a boom or slump builds up, business cycles become less pronounced. The bottom panel reveals that the aggregated intervention volume is not very stable. For instance, $D$ may increase or decrease for an extended period of time.

= = = = = **Figure 2 goes about here** = = = = =

Let us now turn to the second policy. Suppose the government determines to offset income trends by deficit spending when income has just been falling and by collecting a budget surplus when income has been rising. Such a trend-offsetting rule may be expressed as

$$G_t^T = g^T (Y_{t-2} - Y_{t-1}), \tag{21}$$

where $g^T > 0$ is again the policy maker's control parameter.

Figure 3 presents the consequences of such a strategy with $g^T = 0.01$ for the evolution of national income. Again, the policy makers are able to reduce the magnitude of business cycles. The main effect of this policy is that it dampens the output trend and thus the destabilizing impact of extrapolative expectations lessens. As revealed in the bottom panel, this time the aggregated intervention volume behaves much more regularly. Furthermore, its maximal amplitude is lower than compared to the other strategy.

= = = = = **Figure 3 goes about here** = = = = =

[8] As in Baumol (1961), we neglect important monetary phenomena such as interest payments.

Overall, one may be tempted to argue that both intervention strategies are useful to reduce output fluctuations. But is such an optimistic conclusion really justified? Already Baumol (1961) remarked that plausible and reasonable contracyclical policies may turn out to be a dangerous tool. In his linear (multiplier-accelerator) framework, trend-offsetting and level-adjusting policies may, contrary to the economists intuition, aggravate fluctuations in economic activity. Let us therefore explore what happens in our nonlinear model if the government uses different control parameters.

Figure 4 contains two bifurcation diagrams for the impact of level and trend interventions on output fluctuations. The control parameters $g^L$ (top panel) and $g^T$ (bottom panel) are increased in 100 discrete steps from 0 to 0.25. For each value of the control parameter, 400 observations are plotted, after discarding a transient phase of 1000 periods. The remaining parameters are as in section 3. Bifurcation diagrams are a powerful graphical instrument, illustrating the dynamic behavior of a system for a wide range of the underlying bifurcation parameters.

What are the results? Roughly speaking, the maximal amplitude tends to decrease as the two bifurcation parameters increase. Unfortunately, there are some dramatic interruptions of this process. Both types of intervention strategies may for some parameter values yield an explosion of the system (these areas are represented by blanks in the bifurcation diagrams). This is, for instance, the case for about $0.05 < g^T < 0.075$. Surprisingly, for $0.075 < g^T < 0.25$ the same strategy completely stabilizes output motion. Similar results are observed for the other strategy. For, e.g., $g^L = 0.23$, the level-adjusting policy eliminates fluctuations in economic activity, yet for, e.g., $g^L = 0.055$ the same strategy triggers an unstable orbit. To sum up, simple and well-intended intervention policies may lead to a more stable economy, they may however also cause the opposite. This should give policy makers a clear warning: Governmental stabilization programs require careful analysis and monitoring.

**= = = = = Figure 4 goes about here = = = = =**

## 5 Routes to business cycles

So far, our focus was on one particular parameter combination. From an empirical point of view, however, we don't have much reliable information about the "true" parameter setting. One exception may be the consumption parameter for which $a = 0.9$ seems to be a reasonable choice. In addition, the parameter $c$ is by definition bounded between 0 and 1. The goal of this section thus is to explore the dynamics of the model (without contracyclical interventions of the government) for a broader range of parameter combinations. Furthermore, we have to take into account different initial conditions since they may also affect the dynamics.

Figure 5 presents two bifurcation diagrams for the parameters $h$ and $d$. In the top panel, the rationality parameter $h$ is increased from 0 to 2 in 100 discrete steps. Visual inspection reveals that when the rationality of the consumers is low, no fluctuations in business cycles appear. However, when $h$ is larger than about 0.8, endogenous dynamics may emerge. Why do we observe such a rational route to business cycles?[9] Remember that the lower the rationality of the agents, the less they recognize differences in the fitness of the two predictors. Hence, at least some agents will always rely on stabilizing regressive expectations and thus the economy may converge towards its near full employment output level. The bottom panel reveals that business cycles may appear independently of how costly regressive expectations are. Even for $d = 0$, endogenous changes in national income may appear.

**= = = = = Figure 5 goes about here = = = = =**

[9] Brock and Hommes (1997) seem to be the first who have observed that an increase in the agents' rationality may lead to endogenous complex dynamics. They call this phenomenon a "rational route to randomness".

Note that for some values of $d$, the data points in figure 5 scatter quite asymmetric. This is a typical sign for coexisting attractors. In fact, figure 6 reveals that for $d = 0.75$ five attractors coexist (the remaining parameters are again as in section 3). Whether the system settles down on the one or the other attractor critically depends on the choice of initial conditions. Note that the dynamics of the attractors may be quite different. From top to bottom we see (i) moderate swings in national income mainly above $\overline{Y} = 10000$, (ii) moderate swings in national income mainly below $\overline{Y} = 10000$, (iii) significant swings in national income mainly below $\overline{Y} = 10000$, (iv) significant swings in national income mainly above $\overline{Y} = 10000$, and (v) convergence towards $\overline{Y} = 10000$. Coexisting attractors may have severe consequences for an economy. For instance, already a tiny exogenous shock may push the economy from a low to a high volatility regime.

**= = = = = Figure 6 goes about here = = = = =**

This phenomenon is further illustrated in figure 7. Here we display the basins of attraction of the five coexisting attractors of the pervious figure for different initial conditions. The initial conditions are varied as follows. In all cases, we set $Y_1$ equal to 10000. However, on the horizontal (vertical) axis, we increase $Y_3$ ($Y_2$) from 9980 to 10020 in 80 discrete steps. The five panels correspond to the five coexisting attractors (in the same order of appearance, i.e. from the top left to the bottom right). The black dots indicate initial value combinations for which trajectories have finally settled on their attractors. Note that a quite intricate picture emerges. Whether the system settles on the one or other attractor may indeed depend on very small differences in the initial conditions.

**= = = = = Figure 7 goes about here = = = = =**

Figure 8 again shows bifurcation diagrams, now for parameters $c$ and $b$ which are increased from 0 to 1 and 0 to 2, respectively. Surprisingly, now clear cut picture emerges. As is visible in the top panel, when *c* is too low or too high, the system is not stable and an explosion may occur (these areas are represented by blanks in the bifurcation diagram). The bottom panel shows that when *b* is low, the system may approach its steady state level. However, when *b* increases, complex dynamics but also unstable trajectories may appear.

**= = = = = Figure 8 goes about here = = = = =**

Let us finally explore for which combinations of *h* versus *d* and *c* versus *b* the system generates stable or unstable trajectories. The left panel of figure 9 shows the basin of attraction for parameter *h* versus *d* which are both increased from 0 to 2 in 80 discrete steps (the remaining parameters are as in section 3). The black dots then indicate parameter combinations which do not trigger explosions. The right panel shows the same for the parameters *c* and *d* which are increased from 0 to 1 and 0 to 2, respectively. Interestingly, independently of parameters *h* and *d*, only none-explosive trajectories are observed. However, this is not the case for parameters *c* and *b*. In particular, if *b* exceeds a certain threshold, the evolution of national income may become unstable. Note that the pattern which emerges is intricate, there also exists some islands of stability.

**= = = = = Figure 9 goes about here = = = = =**

## 6 Conclusions

Our aim is to study the impact of heuristic expectation formation on fluctuations in economic activity when boundedly rational agents may select between competing predictors. Within our model, agents have the choice between simple and cheap extrapolative and sophisticated, yet costly regressive expectations. The agents prefer rules which possess a high fitness, i.e. produce low squared prediction errors. Since competition between the predictors introduces a nonlinearity, complex fluctuations in economic activity may emerge.

At least at first sight, this may give policy makers an opportunity to stabilize the economy. And indeed, for certain specifications of the intervention strategies, output variability declines. Unfortunately, life is more complicated, especially in a nonlinear world. Common policies such as trend offsetting or level-adjusting interventions turn out to be a mixed blessing. If the policy makers pick the wrong intervention strength – and here tiny differences may already matter – output fluctuations are amplified. This again demonstrates that it is important to understand the causes of business cycles.

Let us finally point out some avenues for future research. First, within our model, agents who form regressive expectations are able to compute the near full employment output level. Given their level of rationality, this may appear as a rather strong assumption. A promising extension may be to consider the case in which boundedly rational agents seek to learn this output level from current and past observations. Second, we ignore important monetary phenomena. It would be interesting to see how private and governmental activity may affect the interest rate and how this, in turn, may feed back on their behavior. We hope that our paper will stimulate further work in this important research direction.

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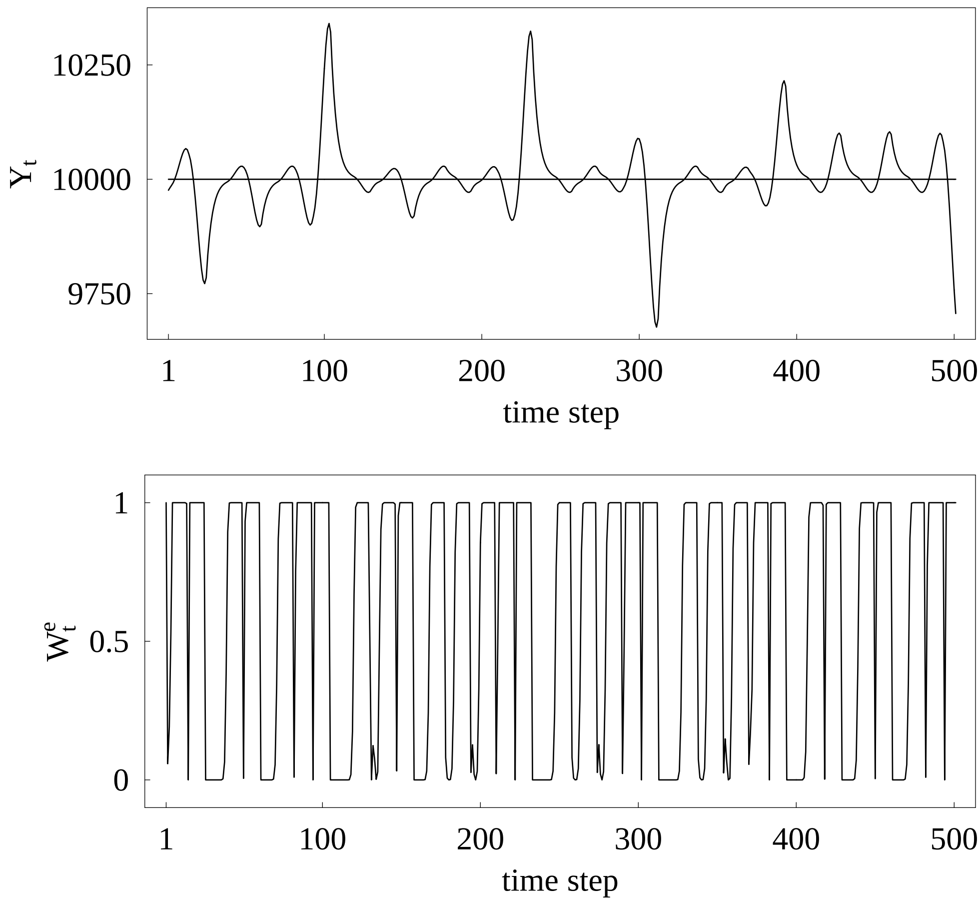


Figure 1: The evolution of national income and the impact of the predictors for 500 observations. Parameter setting as in section 3.

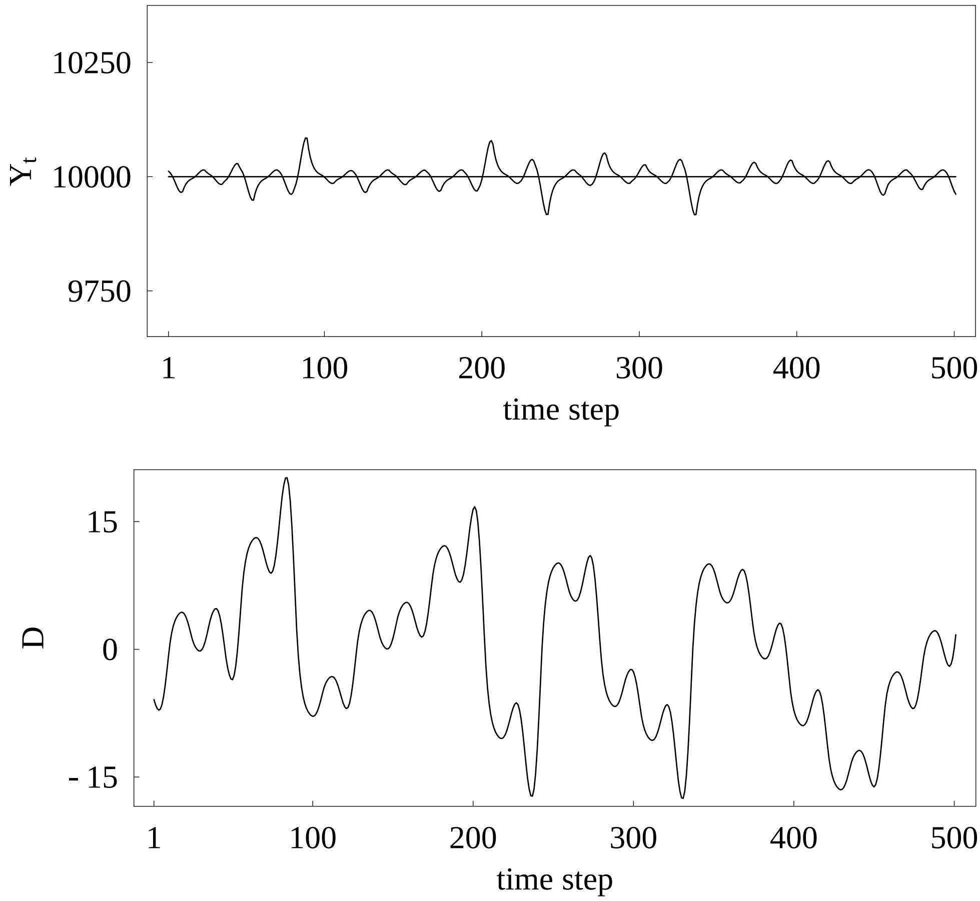


Figure 2: The evolution of national income and accumulated governmental interventions for 500 observations. Parameter setting as in section 3, but $g^L = 0.05$ .

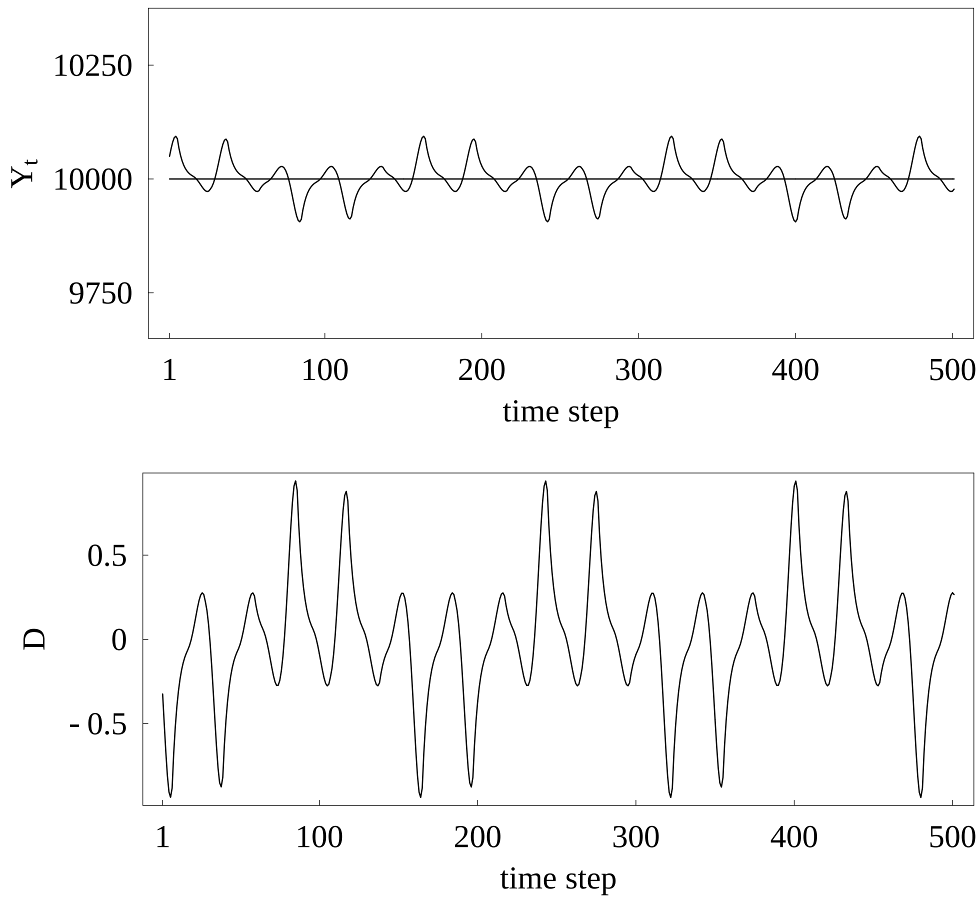


Figure 3: The evolution of national income and accumulated governmental interventions for 500 observations. Parameter setting as in section 3, but $g^T = 0.01$ .

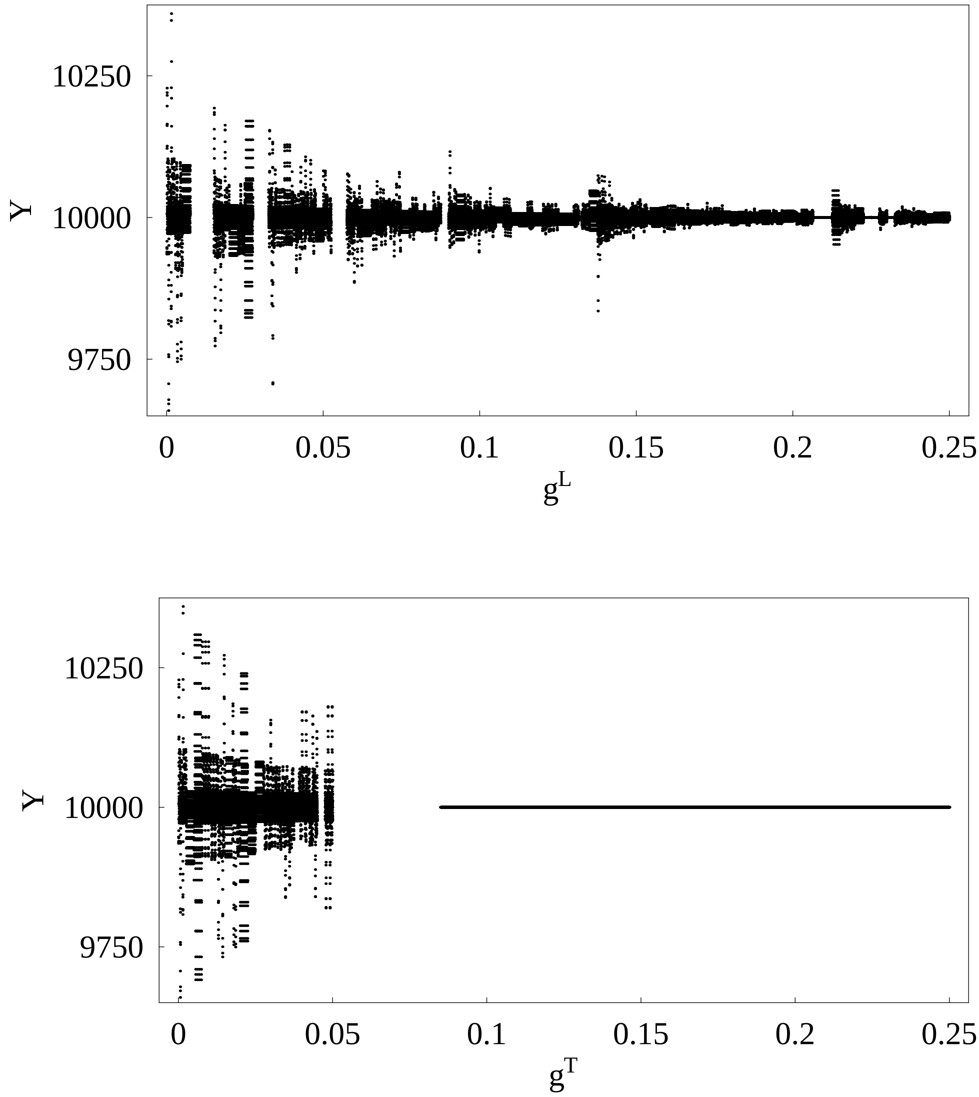


Figure 4: Bifurcation diagrams for the impact of level and trend interventions on output fluctuations. The parameters $g^L$ and $g^T$ are increased in 100 discrete steps from 0 to 0.25. For each parameter value, 400 observations are plotted. We allow for a transient phase of 1000 periods. The remaining parameters are as in section 3.

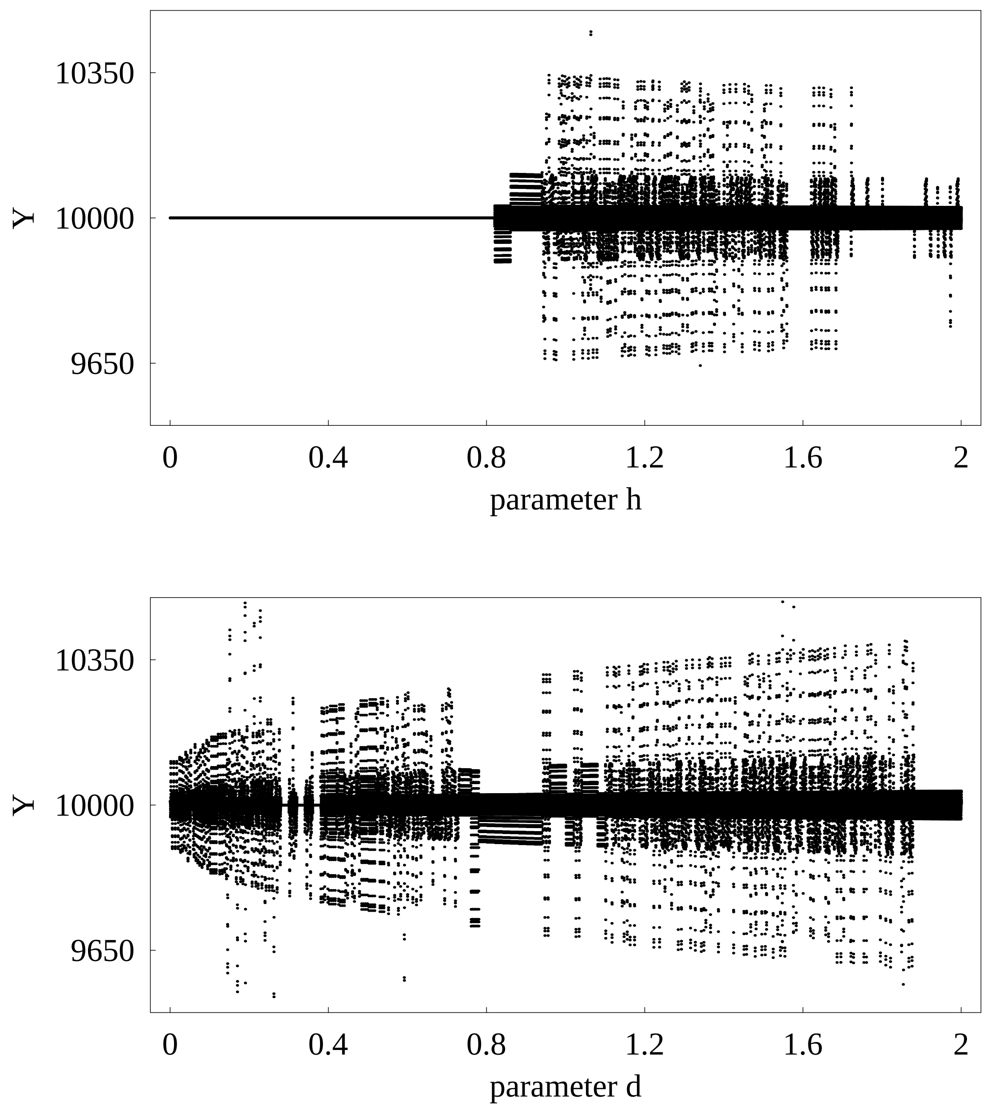


Figure 5: Bifurcation diagrams for the parameters *h* and *d* which are increased in 100 discrete steps from 0 to 2. For each parameter value, 400 observations are plotted. We allow for a transient phase of 1000 periods. The remaining parameters are as in section 3.

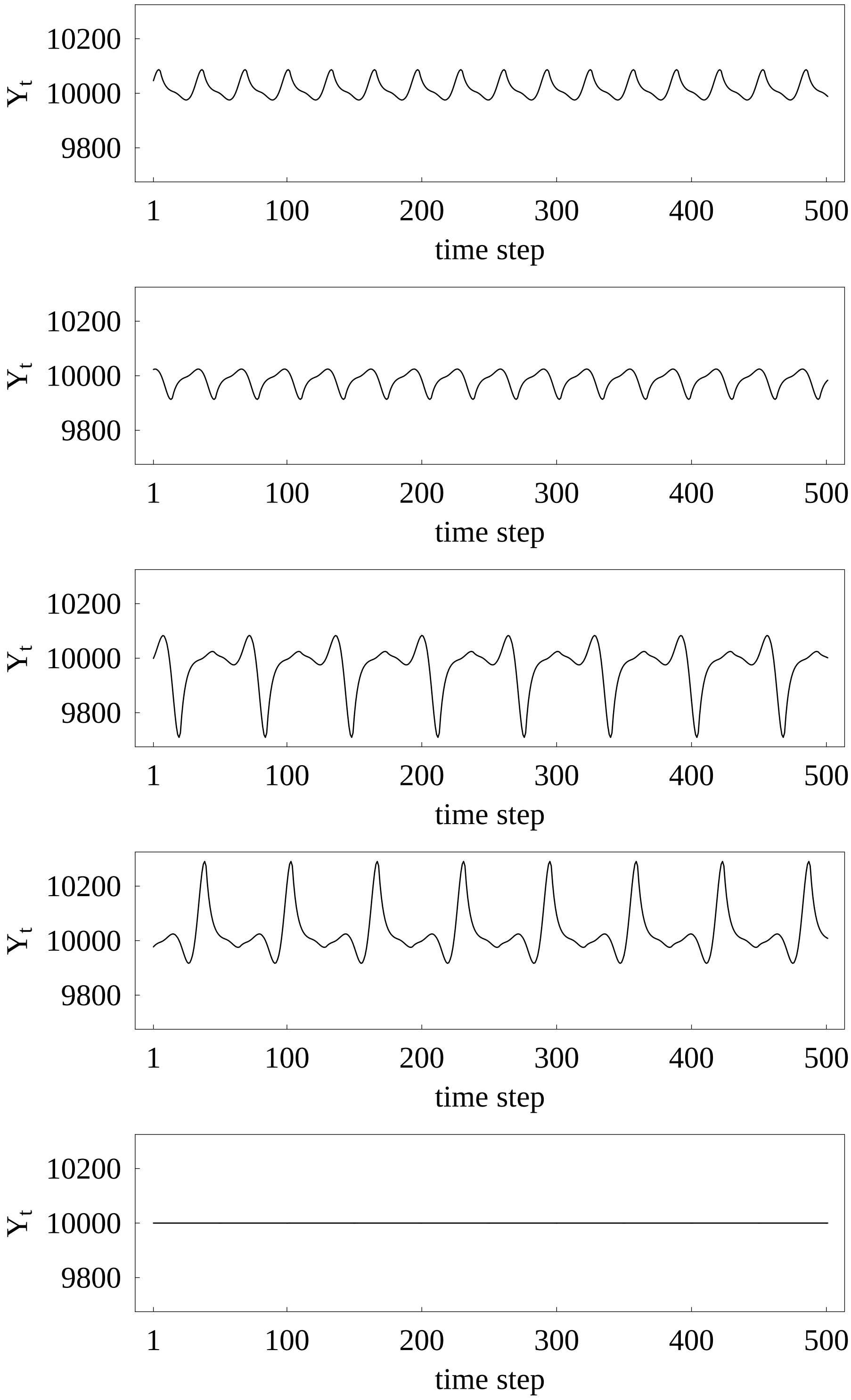


Figure 6: The panels show five coexisting attractors of the national income variable in the time domain for $a = 0.9$, $b = 1.4$, $c = 0.15$, $d = 1.2$, $h = 1$, $\bar{I} = 1000$ and different initial values $Y_1$, $Y_2$ and $Y_3$.

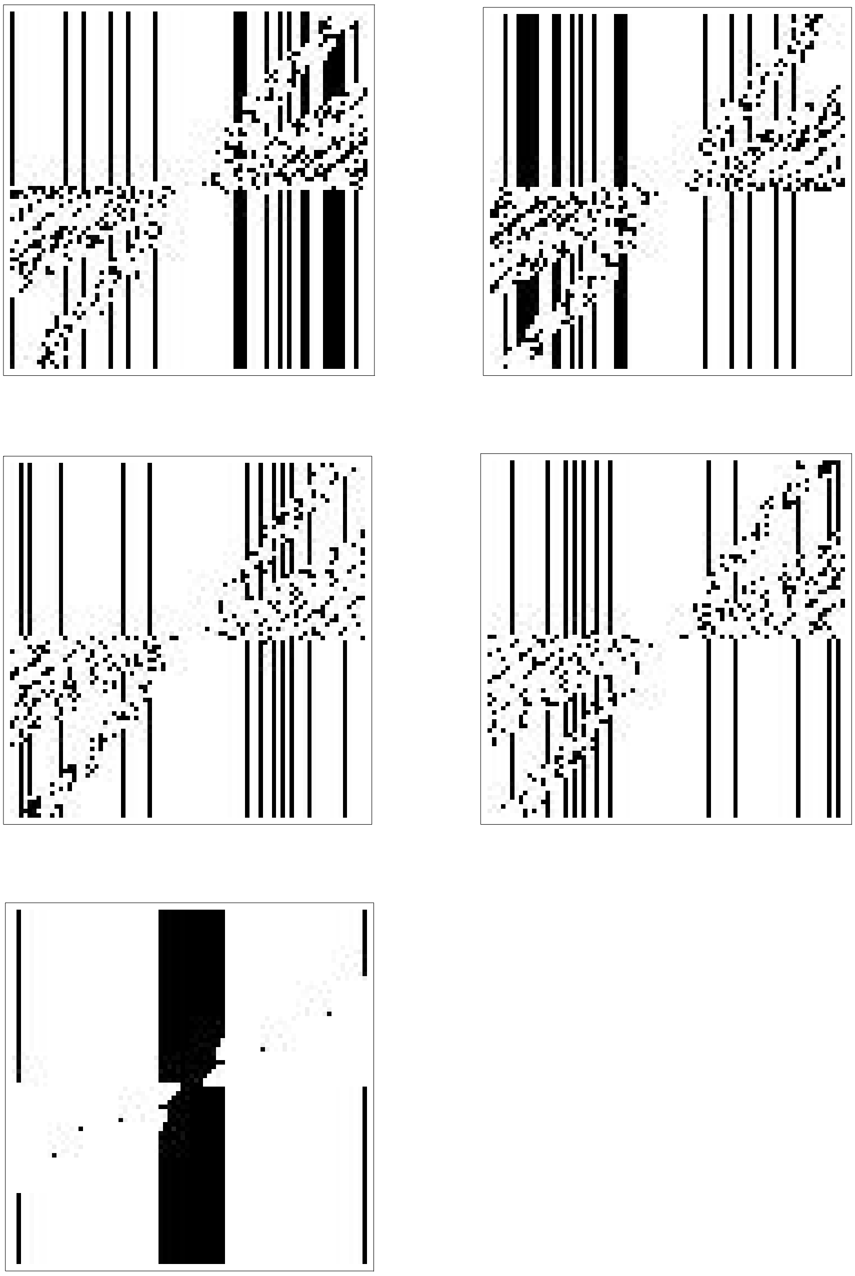

Figure 7: The panels show basins of attraction for the five coexisting attractors of figure 6. On the horizontal (vertical) axis, $Y_3$ $(Y_2)$ is increased from 9980 to 10020 in 80 discrete steps. $Y_1$ is set to 10000. The black dots indicate initial value combinations which lead to the coexisting attractors displayed in figure 6.

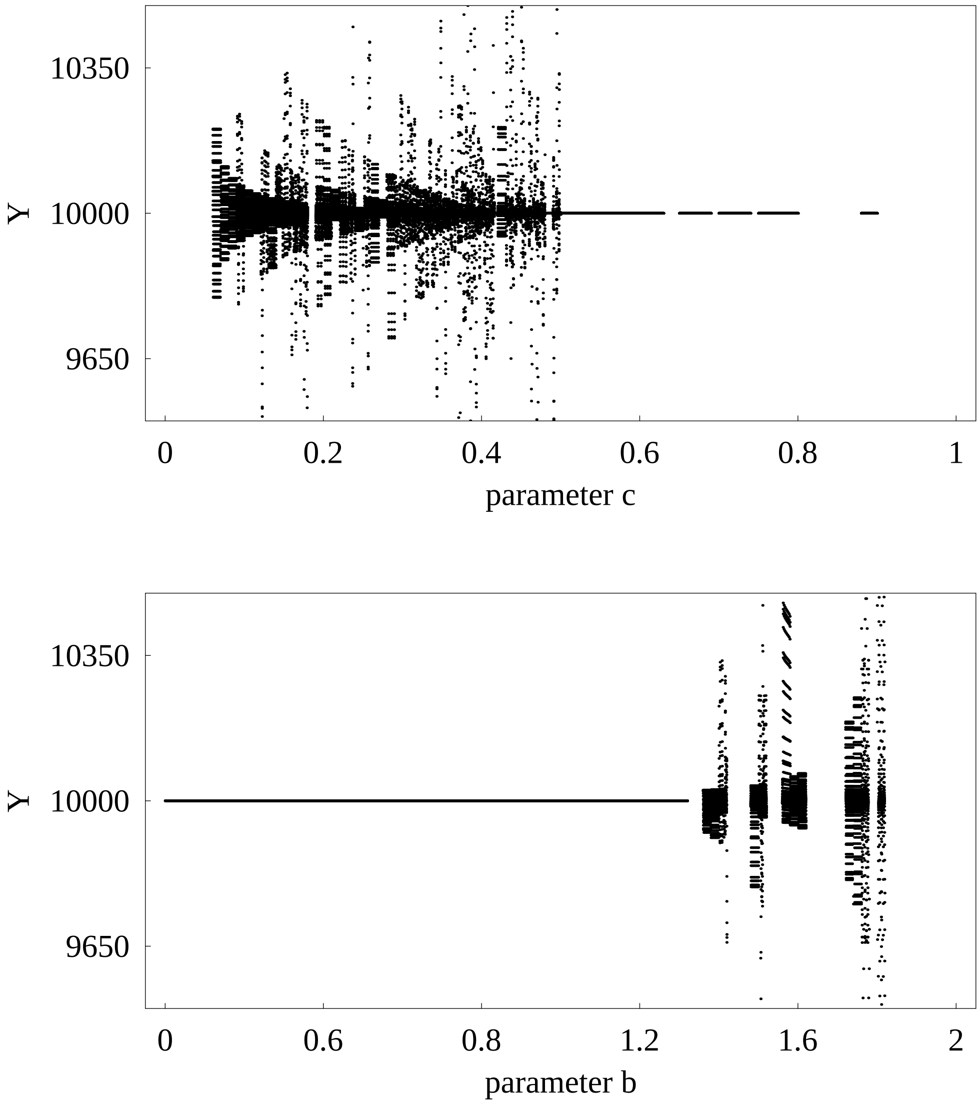


Figure 8: Bifurcation diagrams for the parameters *c* and *b* which are increased in 100 discrete steps from 0 to 1 and 0 to 2, respectively. For each parameter value, 400 observations are plotted. We allow for a transient phase of 1000 periods. The remaining parameters are as in section 3.

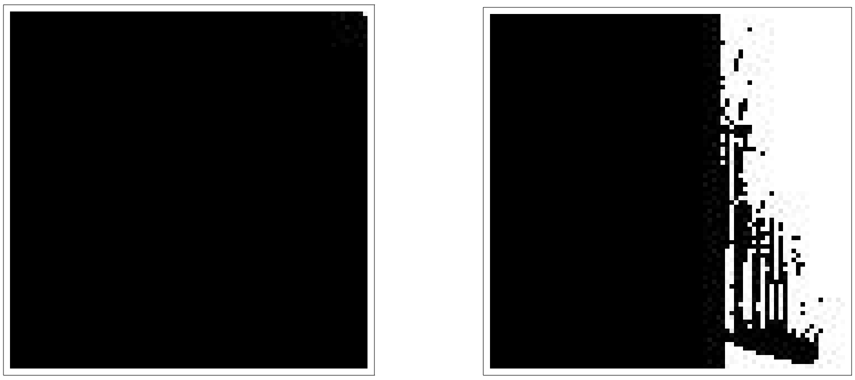

Figure 9: Basins of attraction for parameters *d* versus *h* (left) and *c* versus *b* (right). The black dots indicate parameter combinations which do not trigger explosions. Parameters *d* and *h* are increased from 0 to 2 in 80 discrete steps. Parameter *c* and *d* are increased from 0 to 1 and 0 to 2, respectively, in 80 discrete steps. The remaining parameters are as in section 3.