# Competition with limited attention to quality differences

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# Competition with limited attention to quality differences*

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**Abstract**

I analyze the implication of consumers' limited attention to quality differences on market outcomes and welfare. I model this limited attention to quality differences with a perception threshold: Consumers only perceive quality differences between goods that exceed the consumers' perception threshold. The model allows for two types of equilibria: equilibria with distinguishable and equilibria with indistinguishable qualities. I show that horizontal product differentiation, which gives firms market power, affects equilibrium selection. If firms are horizontally differentiated, firms produce goods with indistinguishable qualities. Then, limited attention harms consumers and benefits firms. In contrast, if firms are not horizontally differentiated, firms produce goods with distinguishable qualities. Then, limited attention has no effect on consumers' welfare or firms' profits.

KEYWORDS: Limited Attention, Perception Threshold, Product Differentiation, Product Quality.
JEL CODES: D43, D91, L13.

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# 1 Introduction

Increasing evidence documents consumers' limited attention to information about goods.[1] When consumers pay only limited attention to goods, consumers might overlook differences between goods and choose inferior goods. The quality of goods is often difficult to compare and consumers pay only limited attention to quality differences. For example, evidence documents limited attention to differences in fuel costs of cars (Allcott, 2013), energy costs of lightbulbs (Allcott and Taubinsky, 2015), coffee quality (Giacalone, Fosgaard, Steen, and Münchow, 2016), or restaurant hygiene (Dai and Luca, 2020). Nevertheless, consumers are not always inattentive to quality differences: If the quality differences are sufficiently large, consumers notice which good has the higher quality.

In this article, I analyze the implications of such limited attention of consumers to quality differences between goods on market outcomes and welfare. If consumers pay only limited attention to quality differences, two types of equilibria can occur: Equilibria with distinguishable qualities and equilibria with indistinguishable qualities. In equilibria with distinguishable qualities, consumers perceive the goods perfectly and choose the utility-maximizing good. In equilibria with indistinguishable but nevertheless different qualities, consumers perceive the goods as having identical quality and, therefore, some consumers might be harmed by buying a good with worse quality than expected.

My objective is to analyze under which conditions firms have an incentive to produce goods with distinguishable and under which conditions firms have an incentive to produce goods with indistinguishable qualities. In particular, I explore whether consumers' attention influences the quality distribution in the market and whether consumers are harmed by imperfectly attending to quality differences between goods. To address this, I analyze a model where two firms compete for consumers who pay limited attention to quality differences. I model such limited attention to quality differences with a perception threshold: Consumers only perceive the quality difference between the two goods, if the quality difference exceeds the consumers' perception threshold.

Limited attention to quality differences harms consumers, if consumers do not notice existing quality differences and buy a good with lower than expected quality. In addition, limited attention to quality differences harms consumers, if firms' investments in quality decrease as a consequence of limited attention. Limited attention reduces the incentives of firms to invest in quality, if they have to fear that their rival undercuts their quality unnoticeably. Then, consumers do not notice the quality difference between the goods and are unwilling to pay a premium for the higher quality. Thus firms' incentives to invest in quality are lower because the firms' investments in quality are not fully rewarded. To capture this undercutting and to analyze the effects on quality provision, I assume that

[1] See, for example, Chetty, Looney, and Kroft (2009); Brown, Hossain, and Morgan (2010); Lacetera, Pope, and Sydnor (2012); Busse, Lacetera, Pope, Silva-Risso, and Sydnor (2013); Englmaier, Schmöller, and Stowasser (2018).

firms choose their qualities sequentially.

I highlight the role of horizontal product differentiation for equilibrium selection.[2] Being horizontally differentiated gives firms market power to set prices above marginal costs for otherwise identical goods. In contrast, without horizontal product differentiation, price competition for identical goods is intense and firms choose prices equal to marginal cost. Therefore, the presence/absence of market power due to horizontal product differentiation affects firms incentives to differentiate on the quality dimension. Firms produce goods with distinguishable qualities strategically to increase market power. If firms have sufficient market power even with indistinguishable qualities because they are horizontally differentiated, producing distinguishable qualities is not necessary.

I start the analysis with a model where firms are horizontally differentiated and incur fixed costs for quality. This captures situations where investing in quality is independent of the quantity that a firm sells; for example, investing in research and development, updating software and hardware, buying more efficient machines, or training employees. I show that firms produce goods with indistinguishable qualities in equilibrium: Both firms want to avoid being the firm with a noticeably lower quality as this leads to lower prices and less demand. To avoid being the firm with noticeably lower quality, the first-mover has to choose its quality high enough to discourage the second-mover from setting a noticeably higher quality. To make the quality difference noticeable, the second-mover would always have to set a quality that exceeds the quality of the first-mover by the perception threshold. If the first-mover already sets a high quality, to make its quality noticeably higher, the second-mover would incur high quality cost.

In the subgame-perfect equilibrium, the second-mover sets the lowest possible quality that is still indistinguishable from the quality of the first-mover. That means, the second-mover free rides: The second-mover benefits from the limited attention of the consumers, who think that the second-mover offers the good at the same quality as the first-mover, without incurring the same quality cost as the first-mover. With decreasing attention, i.e., with increasing perception thresholds, the quality that discourages the second-mover from producing goods with noticeably higher quality decreases. Thus with decreasing attention, both firms reduce their quality until both firms produce zero quality.

The horizontal product differentiation ensures that firms can set prices above marginal cost and make positive profits even if they produce goods with indistinguishable qualities. As firms produce goods with indistinguishable qualities, firms choose the same prices, split the market equally, and receive the same revenue. As the first-mover produces higher quality, it incurs higher cost. Consequently, the first-mover makes less profit and has a *first-mover disadvantage*. As with decreasing attention firms choose (weakly) lower

[2] I compare the cases where firms are horizontally differentiated and where firms are not horizontally differentiated. Alternatively, one could think of this as consumers paying attention to the horizontal characteristics or not paying attention to the horizontal characteristics of the goods.

quality, quality cost are (weakly) decreasing. Thus as revenues are constant, firms' profits (weakly) increase with decreasing attention. In contrast, as firms sell at the same price and consumers are harmed by lower quality, consumer surplus (weakly) decreases with decreasing attention. Overall, the increasing profits of firms do not balance the decreasing surplus of consumers such that overall welfare (weakly) decreases with decreasing attention.

If the firms are not horizontally differentiated, firms produce goods with distinguishable qualities in equilibrium. If firms produce goods with indistinguishable qualities, consumers perceive the goods as identical and the intense price competition ensures that firms choose prices equal to marginal cost. As firms have fixed costs for quality, if firms produce goods with positive quality, this leads to negative profits. Consequently, firms have an incentive to produce goods with distinguishable qualities. By setting a sufficiently high quality in the first stage, the first-mover can ensure that it is the firm with the noticeably higher quality and realize a higher profit. Then, the second-mover responds with a noticeably lower quality. The first-mover thus has an advantage if firms are not horizontally differentiated. Furthermore, in contrast to horizontally differentiated firms, as firms always maximally differentiate, firms' qualities are independent of the perception threshold. That means, independent of how attentive consumers are, firms always choose the same qualities and prices. The attentiveness of consumers thus also has no effect on producer surplus, consumer surplus, and welfare. Consequently, the absence of horizontal product differentiation changes firms' incentives: Firms need to differentiate noticeably in the quality dimension to create market power and realize positive profits.

To analyze how the results depend on the cost for quality, I analyze a variant of the model with marginal costs in Section 6. Assuming marginal cost of quality captures situations where investments in quality depend on the quantity that a firm sells; for example, producing goods with more expensive ingredients. Compared to the fixed cost case, with marginal cost, horizontal product differentiation has a limited effect on the resulting equilibria. With and without horizontal product differentiation, an equilibrium with indistinguishable qualities always exists. In addition, an equilibrium with distinguishable qualities may exist. Without horizontal product differentiation, an equilibrium with distinguishable qualities exists for a larger range of values. The differences between the marginal and the fixed costs case stem from the effects on price competition. In the marginal cost case, producing indistinguishable qualities is more attractive, because firms differ in their marginal costs and account for their quality costs in their prices.

The remainder of this article is structured as follows: Section 2 describes the contribution to the related literature. Section 3 introduces the model. Section 4 derives the results for the fixed cost case with horizontal product differentiation. Section 5 derives the results for the fixed cost case without horizontal product differentiation. In Section 6, I compare the results to a model with marginal costs of quality. Section 7 concludes.

## 2 Related literature

An increasing literature analyzes the implications of consumers' limited attention on market outcomes and welfare.[3] One strand of this literature analyzes the implications when limited attention prevents consumers from noticing all available goods in a market such that firms have to compete for consumers' attention before competing for their business (Eliaz and Spiegler, 2011a,b; Haan and Moraga-González, 2011; de Clippel, Eliaz, and Rozen, 2014; Manzini and Mariotti, 2018; Astorne-Figari, López, and Yankelevich, 2019; Armstrong and Vickers, 2022). Another strand of this literature analyzes the implications when limited attention prevents consumers from (perfectly) perceiving all characteristics of the goods that they consider: for instance, the quality (Armstrong and Chen, 2009) or the add-on costs of goods (Gabaix and Laibson, 2006; Heidhues, Köszegi, and Murooka, 2016, 2017). I focus on the implications of limited attention to quality differences and thus contribute primarily to the second strand of this literature.

In particular, I model limited attention to quality differences with a perception threshold such that consumers notice large quality differences that exceed the perception threshold, but do not notice small quality differences that are below the perception threshold. The assumption that individuals perceive similar options as identical proves troublesome for the transitivity of the indifference relation. For example, Luce (1956) and Rubinstein (1988) provide decision-making models that account for such constraints on similarity perception. Yet, coarse perception is not necessarily detrimental. Horan, Manzini, and Mariotti (2022) identify conditions under which coarser perception in a model with noisy perception might even improve choice.

Current models on decision-making under limited attention also account for the influence of similarity on choice. Bordalo, Gennaioli, and Shleifer (2012, 2013) and Köszegi and Szeidl (2013) analyze the implications of salience on decision-making. The salience of a dimension depends on how similar options are in this dimension. For options with multiple dimensions (e.g., lotteries with various outcomes or goods with different payment plans), the more similar options are in one dimension, the less weight this dimension receives in the evaluation.[4] In contrast, in this article, I focus on the interactions of firms that face consumers with limited attention.

I contribute primarily to the literature on market interactions with boundedly rational consumers; specifically, to models where consumers only notice differences between goods that exceed their perception threshold (Allen and Thisse, 1992; Bachi, 2016; Webb, 2017; Balart, 2021; Chung, Liu, and Lo, 2021). Allen and Thisse (1992) and Bachi (2016) analyze price competition in duopolies where consumers' perception of prices is subject to a perception threshold. If the prices are too similar, consumers perceive them as identical.

[3] See Gabaix (2019) for an overview of the limited attention research.

[4] See Bordalo, Gennaioli, and Shleifer (2016) for an application to market competition.

Both models show that consumers' perception thresholds lead to prices above marginal cost and positive profits. In other words, the imperfect perception of consumers allows firms to overcome the Bertrand paradox. Balart (2021) analyzes the consequences of a perception threshold in a model of horizontal product differentiation. Balart (2021) shows that firms differentiate more under limited than under full attention and that more inattention to the horizontal characteristics of the goods might lead to higher profits for the firms. Chung, Liu, and Lo (2021) analyze situations where consumers cannot detect small utility differences between two options. Then, adding a third option that is noticeably different from one but not both options helps consumers to infer the better deal.

Webb (2017) analyzes firms' strategic interactions when consumers have a relative perception threshold about quality differences. Although, I also study a model with limited attention to quality differences, the models differ significantly in the setup as well as in the results. Webb (2017) focuses on a relative perception threshold. In contrast, I focus on an absolute perception threshold. A relative perception threshold is often justified by Weber's Law.[5] Yet, a relative perception threshold is only sensible for quality values that are sufficiently large. With one firm producing zero quality, even if the rival firm produces goods with extremely low quality, the relative difference is infinite. Thus all consumers notice the quality difference between the goods—even if they have a high perception threshold. This logic explains why Weber's law does not hold at the extremes (see, e.g., Hunt, 2007). Consequently, I focus on the implications of an absolute perception threshold (see also Bachi, 2016; Chung, Liu, and Lo, 2021; Balart, 2021). Thereby, I capture new insights into the incentives of firms and allow for equilibria with indistinguishable but low qualities. In contrast, in Webb (2017), firms always produce goods with distinguishable qualities. In addition, in Webb (2017), profits and consumer surplus depend on the perception threshold, whereas, I show that qualities, profits, and consumer surplus do not always depend on the perception threshold.

Overall, I contribute to the literature by contrasting results with and without horizontal product differentiation. Thereby, I provide a better understanding of the conditions under which equilibria with indistinguishable qualities exist. In particular, I highlight the role of market power. Firms produce distinguishable qualities strategically to increase market power. If firms have sufficient market power even with indistinguishable qualities, producing distinguishable qualities is not necessary.

[5] Weber's Law states that the difference between two stimuli which is just noticeable depends on the overall level of the stimuli (Hunt, 2007).

# 3 Model

Consider two horizontally differentiated firms, firm A and firm B, that compete in qualities and prices for a unit mass of consumers. I follow Hotelling (1929) in modeling horizontal product differentiation as a real line $[0, 1]$ with firm A located at 0 and firm B located at 1. In Section 5, I derive the results for goods that are not horizontally differentiated and discuss how the results depend on the assumption of horizontally differentiated firms.

Consumers are uniformly distributed on $[0, 1]$. The position $x \in [0, 1]$ of a consumer denotes the consumer's ideal version of the good. Consumers buy exactly one unit of the good. By buying that unit from firm $i \in \{A, B\}$, the consumer at position $x$ receives utility

$$u_x(i) = v + q_i - p_i - (x - y_i)^2, \tag{1}$$

where $v \gg 0$ is the gross utility of the good, $q_i \in [0, 1]$ is the quality, $p_i \in \mathbb{R}_0^+$ the price, and $y_i$ the location of firm $i$ in the product space. I assume that $v$ is large enough such that consumers always buy the good in equilibrium. I follow d'Aspremont, Gabszewicz, and Thisse (1979) in modeling disutility from consuming a non-ideal good as quadratic: $(x - y_i)^2$.

Consumers are constrained in their perception of quality. Consumers only perceive quality differences between the goods if the quality difference is sufficiently large. The perceived quality $\hat{q}_i$ is thus

$$\hat{q}_i = \begin{cases} q_i & \text{if } q_B < q_A - \tau \\ q(q_A, q_B) & \text{if } q_A - \tau \leq q_B < q_A + \tau \\ q_i & \text{if } q_B \geq q_A + \tau, \end{cases} \tag{2}$$

where $q(q_A, q_B) \in [0, 1]$ and the perception threshold is $\tau \in (0, 1)$.[6] The perception threshold $\tau$ is identical for all consumers. The perception threshold captures the consumers' limited attention to quality differences. The larger the perception threshold, the more inattentive consumers are. The model includes perfect perception as the limiting case $\tau = 0$. If the quality difference is larger than the perception threshold $\tau$, consumers

[6]Expression (2) captures:

$$\hat{q}_i = \begin{cases} q_i & \text{if } |q_A - q_B| > \tau \\ q(q_A, q_B) & \text{if } |q_A - q_B| < \tau. \end{cases}$$

That is, if the quality difference exceeds the perception threshold, i.e., $|q_A - q_B| > \tau$, the true quality is observed and if the quality difference is below the perception threshold, i.e., $|q_A - q_B| < \tau$, the consumers observe both qualities as identical. Because of the discontinuities of profits at $|q_A - q_B| = \tau$, I have to make choices such that an equilibrium exists. Assuming that the consumers do not notice a quality difference if $q_A - \tau \leq q_B < q_A + \tau$ but notice a quality difference if $q_B < q_A - \tau$ or if $q_B \geq q_A + \tau$ ensures this.

perceive the quality of each firm perfectly. If the quality difference is smaller than the perception threshold $\tau$, the consumers perceive the quality of firm $i$ as $\hat{q}_i = q(q_A, q_B)$ for both $i \in \{A, B\}$. In other words, the consumers perceive the quality of firm A and the quality of firm B as identical. For instance, $q(q_A, q_B)$ could be the average of $q_A$ and $q_B$. Yet, for the analysis it is not necessary to specify $q(q_A, q_B)$.

For the consumption decision, consumers use the perceived utilities $\hat{u}_x(A)$ and $\hat{u}_x(B)$ instead of the true utilities $u_x(A)$ and $u_x(B)$ given in (1). The perceived utilities differ from the true utilities only in the quality dimension: Instead of the true qualities consumers take the perceived qualities given in (2) into account. A consumer is indifferent between buying from firm A and firm B if

$$\hat{u}_x(A) = \hat{u}_x(B) \Leftrightarrow v + \hat{q}_A - p_A - x^2 = v + \hat{q}_B - p_B - (1-x)^2.$$

Denote the indifferent consumer by

$$\bar{x} \equiv \frac{1 + \hat{q}_A - \hat{q}_B + p_B - p_A}{2}.$$

Then, all consumers $x \leq \bar{x}$ buy from firm A and all consumers $x > \bar{x}$ buy from firm B. As long as $\bar{x} \in (0, 1)$, both firms capture some demand. However, if firms choose sufficiently different prices and/or qualities, one firm may capture the demand of all consumers.

Firms play a three-stage game, depicted in Figure 1: In the first stage, firms observe the perception threshold and, then, firm A chooses its quality $q_A$. In the second stage, firm B observes the quality of firm A and chooses its quality $q_B$. In the third stage, firm A observes the quality of firm B and both firms, independently and simultaneously, set prices. Subsequently, consumers perceive the goods and buy either from firm A or from firm B.

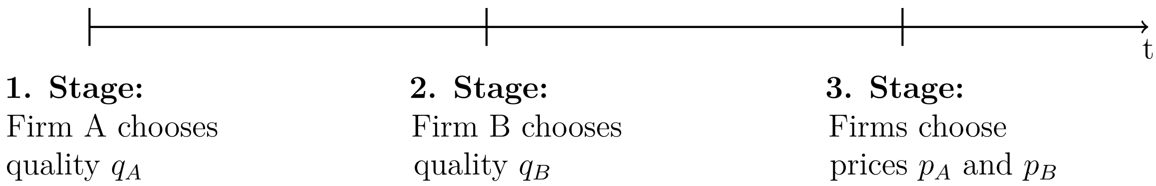


Figure 1: Timeline.

As firm B chooses its quality after observing the quality of firm A, firm B decides whether the quality difference is noticeable. That means, if firm B wants to make its quality indistinguishable from the quality of firm A, firm B has to choose a quality $q_B \in [q_A - \tau, q_A + \tau)$. In contrast, if firm B wants to make its quality distinguishable from the quality of firm A, firm B has to choose a quality $q_B \in [0, q_A - \tau) \cap [q_A + \tau, 1]$.

I assume identical cost functions $C(q_i) = \frac{1}{2}q_i^2$ for both firms.[7] That means, the cost for quality do not depend on the quantity that the firms sell. In Section 6, I contrast this case with the results when costs for quality depend on the quantity.

# 4 Results with horizontal product differentiation

I solve the game by backward induction looking for the pure-strategy subgame-perfect equilibria. In the price-setting stage, firms simultaneously set prices to maximize the following profits, which depend on the qualities set in the first stage and in the second stage[8]

$$\begin{aligned}\Pi_A(p_A, p_B, q_A, q_B) =& p_A\bar{x} - \frac{1}{2}q_A^2\\ \Pi_B(p_A, p_B, q_A, q_B) =& p_B(1-\bar{x}) - \frac{1}{2}q_B^2.\end{aligned}$$

The best replies are

$$\begin{aligned}p_A^*(p_B) &= \frac{1}{2}(\hat{q}_A - \hat{q}_B + p_B + 1)\\ p_B^*(p_A) &= \frac{1}{2}(\hat{q}_B - \hat{q}_A + p_A + 1).\end{aligned}$$

The resulting equilibrium prices are

$$p_A^* = 1 + \frac{\hat{q}_A - \hat{q}_B}{3} \qquad \text{and} \qquad p_B^* = 1 + \frac{\hat{q}_B - \hat{q}_A}{3}.$$

The prices depend on the consumers' perception of quality. Consumers are only willing to pay for a quality difference that they perceive. If $q_A - \tau \leq q_B < q_A + \tau$, i.e., if the qualities are so similar that consumers do not notice the difference, consumers think the firms offer goods with identical quality. Therefore, they are not willing to pay a mark-up for quality. Consequently, in equilibrium, both firms set identical prices. However, as the goods are horizontally differentiated, which reduces price competition, firms can charge prices above marginal cost (here above zero). The resulting price equilibrium is $p_A^* = 1$ and $p_B^* = 1$.

If $q_B \in [0, q_A - \tau, ) \cap [q_A + \tau, 1]$, i.e., if the qualities are sufficiently different so that consumers notice the quality difference, the resulting price equilibrium is $p_A^* = 1 + (q_A - q_B)/3$ and $p_B^* = 1 + (q_B - q_A)/3$. The noticeable quality difference allows the firm with the higher quality to charge a higher price than its competitor.

[7] I adopt this specific cost function to keep the model tractable. Allowing for a more general cost function complicates the analysis substantially.

[8] If firms choose sufficiently different prices, the demand for the good of one firm could become zero. However, in equilibrium, both firms choose prices such that both firms receive some demand.

In the second stage, firm B chooses its quality to maximize its profit taking the prices into account and given the quality choice of firm A. As the prices depend on the quality difference between the goods, firm B's profit is

$$\begin{aligned}\Pi_B(q_A, q_B) =& \frac{1}{2}\left(1 + \frac{\hat{q}_B - \hat{q}_A}{3}\right)^2 - \frac{1}{2}q_B^2 \\ =& \begin{cases} \frac{1}{2}\left(1 + \frac{q_B - q_A}{3}\right)^2 - \frac{1}{2}q_B^2 & \text{if } q_B < q_A - \tau \\ \frac{1}{2} - \frac{1}{2}q_B^2 & \text{if } q_A - \tau \leq q_B < q_A + \tau \\ \frac{1}{2}\left(1 + \frac{q_B - q_A}{3}\right)^2 - \frac{1}{2}q_B^2 & \text{if } q_A + \tau \leq q_B. \end{cases}\end{aligned} \tag{3}$$

For a given quality of firm A, firm B decides whether to produce indistinguishable or distinguishable quality. If firm B chooses a quality that is indistinguishable from the quality of firm A, i.e., $q_A - \tau \leq q_B < q_A + \tau$, no consumer notices a quality difference. As firms then choose identical prices $p_A^* = p_B^* = 1$ in the price-setting stage, firms split the demand equally. That means, firm B makes the same revenue with any $q_B \in [\max\{0, q_A - \tau\}, q_A + \tau)$. But, firm B's cost are increasing in its quality. Thus firm B chooses the lowest quality such that consumers do not perceive the quality difference; this quality is $q_B = \max\{0, q_A - \tau\}$. In other words, it is impossible that firm A has an unnoticeably lower quality than firm B.

If firm B chooses a quality such that $q_B < q_A - \tau$ or $q_B \geq q_A + \tau$, all consumers notice the quality difference. Then, the prices and, consequently, also the profits depend on the quality difference. Overall, firm B maximizes over (3) to derive the best response to the quality of firm A. For a detailed derivation of the best reply, see Appendix A.

In the first stage, firm A chooses its quality taking the subsequent decision of firm B into account. As firm A can never produce unnoticeably lower quality than firm B, firm A is either the firm with noticeably higher, noticeably lower, or unnoticeably higher quality. To avoid being the firm with noticeably lower quality, firm A has to choose a sufficiently high quality in the first stage. Then, firm B has to exceed the quality of firm A by the perception threshold $\tau$ to make the quality difference noticeable. If the quality of firm A is already high, this implies too high cost for firm B. Thus in the subgame-prefect equilibrium, firm A provides a quality that is high enough to discourage firm B from noticeably overbidding firm A's quality. The necessary quality for this depends on the size of the perception threshold. The lower the perception threshold, the higher the necessary quality. As quality is costly, firm A prefers the lowest such quality to discourage firm B from noticeably overbidding its quality. Thus the quality of firm A (weakly) decreases with the perception threshold. As a best response, firm B undercuts the quality of firm A such that the quality difference is just not noticeable, i.e., $q_B = \max\{0, q_A - \tau\}$. Thus also the quality of firm B (weakly) decreases with increasing perception thresholds. Proposition 1 characterizes the resulting subgame-perfect equilibria.

**Proposition 1** *Let*

$$\bar{\tau} \equiv \sqrt{6/55} - 1/2\sqrt{3\sqrt{6/55} - 61/110} \approx 0.00002.$$

*(i)* ***Equilibrium with distinguishable qualities:***
*For $\tau \leq \bar{\tau}$, a pure-strategy subgame-perfect equilibrium exists, where firms choose distinguishable qualities $q_A^* = 21/55$ and $q_B^* = 18/55$ with prices $p_A^* = 56/55$ and $p_B^* = 54/55$.*

*(ii)* ***Equilibrium with indistinguishable qualities:***
*For $\tau \geq \bar{\tau}$, a pure-strategy subgame-perfect equilibrium exists, where firms produce goods with indistinguishable qualities and identical prices $p_A^* = p_B^* = 1$.*

The proof is in Appendix A. Figure 2 illustrates the equilibrium qualities as a function of the perception threshold $\tau$.

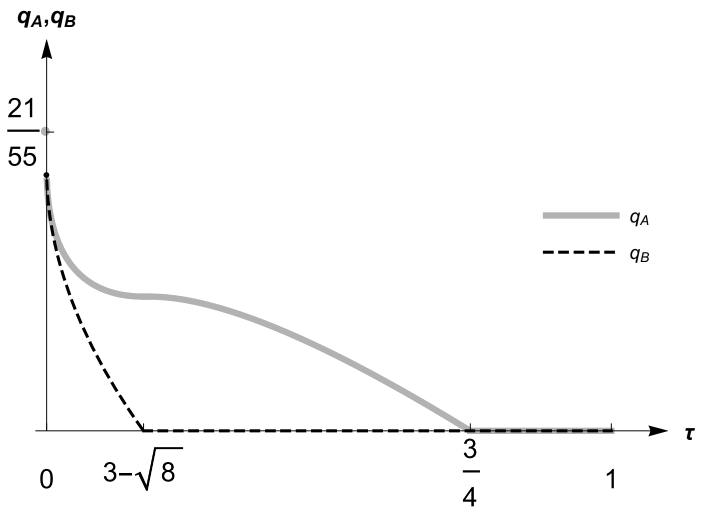


Figure 2: Equilibrium qualities of firm A (solid) and firm B (dashed) as a function of the perception threshold $\tau$.

In the benchmark case under perfect perception, i.e., $\tau = 0$, both firms produce goods with positive quality: $q_A^* = 21/55$ and $q_B^* = 18/55$. Firm A has a first-mover advantage. Firm A takes the best reply of firm B, which is here $q_B^*(q_A) = (3 - q_A)/8$, into account. That means by increasing its quality, firm A has a direct and a strategic effect: Firm A directly increases its profit if it increases its quality. In addition, if firm A increases its quality, firm B reduces its quality which, in equilibrium, also increases firm A's profit. Therefore, in equilibrium, the sequential game structure results in firm A producing goods with higher quality than firm B. This subgame-perfect equilibrium also results for extremely low levels of the perception threshold $\tau$, i.e., for all $\tau \leq \bar{\tau}$. Consumers notice this quality difference which allows firm A to set a higher price than firm B. In addition, because of the quality difference more consumers buy from firm A and firm A makes a higher profit than firm B. Thus for $\tau \leq \bar{\tau}$, firm A has a first-mover advantage.

In contrast, for $\tau \geq \bar{\tau}$, it becomes profitable for firm B to undercut the quality of firm A unnoticeably. That is, firms set qualities such that consumers do not notice the quality difference. Then, the firms set identical prices and split the demand equally. Yet, for $\tau < 3/4$, firm A produces goods with strictly higher quality than firm B: By setting a sufficiently high quality, firm A wants to discourage firm B from producing a noticeably higher quality than firm A. As a consequence, the best reply of firm B is to unnoticeably undercut the quality of firm A. As the perception threshold $\tau$ increases, firm A can reduce its quality without firm B producing noticeably higher quality. Firm B, which produces $q_B^* = \max\{0, q_A - \tau\}$, then also reduces its quality until $q_B^* = 0$. Firm A reduces its quality further with increasing perception threshold $\tau$ until, for $\tau \geq 3/4$, both firms produce zero quality.

Overall, firms produce goods with indistinguishable qualities for all $\tau > \bar{\tau} \approx 0.00002$ and with increasing perception threshold $\tau$ both firms reduce their quality. Figure 3 illustrates the subgame-perfect equilibrium profits of firm A and firm B as a function of the perception threshold $\tau$. Figure 3 shows that firm A makes less profit than firm B for all $\bar{\tau} < \tau < 3/4$. Because the quality difference is unnoticeable, both firms sell at the same price and split the market equally. Thus both firms receive the same revenue, but firm A has higher quality cost. Consequently, firm A makes less profit than firm B and has a first-mover disadvantage. As $\tau$ increases firms A and B reduce their quality until at $\tau \geq 3/4$ both firms produce zero quality. Then, as the firms receive the same revenue and have zero quality cost, firms make the same profits.

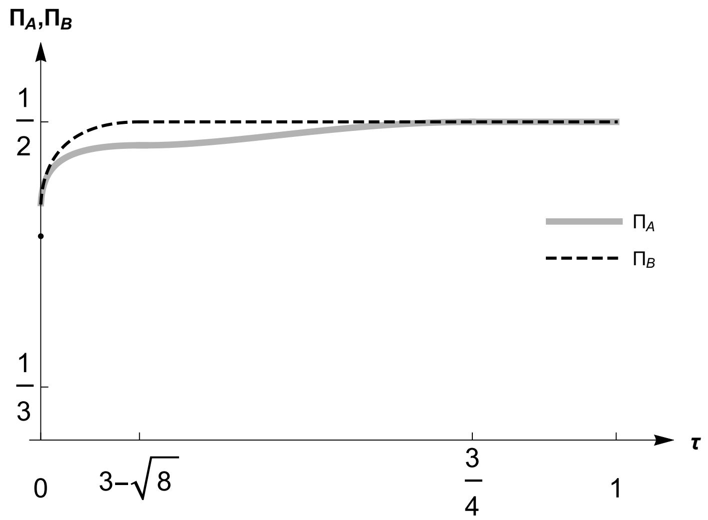


Figure 3: Equilibrium profits of firm A (solid) and firm B (dashed) as a function of the perception threshold $\tau$.

For $\tau > \bar{\tau}$, with increasing $\tau$, firms (weakly) reduce quality. As consumers buy exactly one unit of the good independent of quality, firms' profits (weakly) increase in $\tau$. Thus the producer surplus (weakly) increases in $\tau$. In contrast, as consumers prefer higher quality to lower quality, consumer surplus (weakly) decreases in $\tau$. In sum, as $\tau$ increases, the gains of the firms do not balance the losses of the consumers and welfare (weakly) decreases in $\tau$. Consumers benefit from the high quality under $\tau \leq \bar{\tau}$, when the qualities are distinguishable. Thus consumer surplus and welfare are highest under close to perfect perception, whereas, firms prefer inattentive consumers. Proposition 2 summarizes the consumer surplus, producer surplus, and welfare results and Figure 4 illustrates the consumer surplus, producer surplus, and the welfare dependent on the perception threshold $\tau$.

**Proposition 2** *The producer surplus reaches its maximum for $\tau \geq 3/4$ where qualities are indistinguishable. The consumer surplus and the welfare reach their maxima for $\tau \leq \bar{\tau}$ where qualities are distinguishable.*

The proof is in Appendix B.

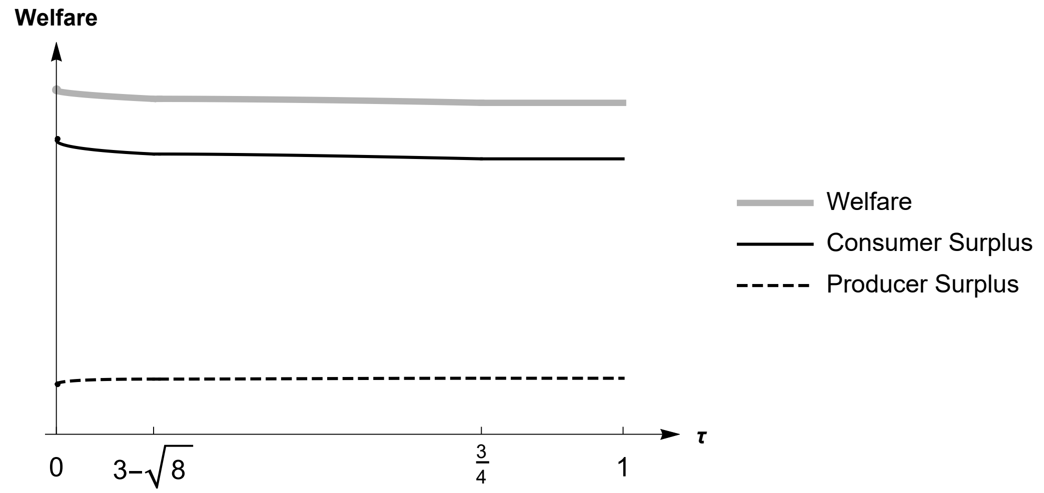


Figure 4: Welfare (solid gray), consumer surplus (solid black), and producer surplus (dashed) as a function of the perception threshold $\tau$ for $v = 6$.

# 5 Results without horizontal product differentiation

In Section 4, when consumers have limited attention, firms produce goods with indistinguishable qualities in the unique subgame-perfect equilibrium. As firms are horizontally differentiated, both firms can still charge prices above marginal cost and make positive profits even if they produce goods with indistinguishable qualities; horizontal product differentiation gives firms market power. In contrast, without horizontal product differentiation, price competition is more pronounced. Then, if firms produce goods with indistinguishable qualities, price competition ensures that firms charge prices equal to marginal cost of zero and make zero revenue. Consequently, it is reasonable to expect that producing goods with distinguishable qualities becomes more attractive and the range of values for which firms produce goods with indistinguishable qualities decreases. In this section, I discuss the resulting subgame-perfect equilibria when firms are not horizontally differentiated and compare the results with the results from Section 4.

Without horizontal product differentiation, consumers' utility and perceived utility from buying the good of firm $i \in \{A, B\}$ become

$$u_x(i) = v + q_i - p_i$$
$$\hat{u}_x(i) = v + \hat{q}_i - p_i.$$

Consumers prefer the good of firm $i \in \{A, B\}$ with $i \neq j$ and $j \in \{A, B\}$ if

$$\hat{u}(i) > \hat{u}(j) \Leftrightarrow p_i < \hat{q}_i - \hat{q}_j + p_j.$$

To sustain the equilibrium, I make the following assumptions about consumers' behavior when they are indifferent between the two firms, i.e., when $\hat{u}(i) = \hat{u}(j)$. First, consumers buy from the firm with the higher quality if the qualities are distinguishable. Second, consumers randomize if the qualities are indistinguishable.[9]

In the price-setting stage, two types of subgames exist. In subgames with indistinguishable qualities, consumers perceive the goods as identical. Consequently, price competition ensures prices equal to marginal cost of zero and zero revenues. In subgames with distinguishable qualities, price competition ensures that the firm with the lower quality chooses prices equal to marginal cost of zero and obtains zero revenue. Yet, the firm with the higher quality charges prices equal to the quality difference and obtains a positive revenue. Thus firms have an incentive to avoid subgames with indistinguishable qualities.

As the first mover, firm A can ensure that it is the firm with the higher profit by setting a sufficiently high quality in the first stage. Then, firm B responds with a noticeably lower quality. As this means that firm B obtains zero revenues, firm B chooses a quality of zero to avoid quality costs. In the subgame-perfect equilibrium, firms thus produce goods with distinguishable qualities, firm B makes zero profit, and firm A has a first-mover advantage and makes positive profits.

Proposition 3 summarizes the results. See Appendix C for a detailed analysis.

**Proposition 3 (Unique equilibrium with distinguishable qualities)**
*When goods are not horizontally differentiated, for all $\tau \in (0, 1)$, in the unique subgame-perfect equilibrium, firms produce goods with distinguishable qualities $q_A^* = 1$ and $q_B^* = 0$ and prices $p_A^* = 1$ and $p_B^* = 0$.*

[9] These tie-breaking rules simply sustain the equilibria. Very similar equilibria occur without these assumptions, because the firm with the higher quality is always able to set a marginally lower price to capture the complete demand.

Comparing the models with horizontal product differentiation and without horizontal product differentiation shows marked differences. The horizontal product differentiation dampens the price competition between the firms and ensures that firms still make sufficient profit if they produce goods with indistinguishable qualities. Therefore, for almost all values of the perception threshold $\tau$, horizontally differentiated firms produce goods with indistinguishable qualities. In contrast, firms that are not horizontally differentiated have no incentive to produce goods with indistinguishable qualities, as indistinguishable qualities imply intense price competition and zero revenues.

Furthermore, in contrast to horizontally differentiated firms, firms' qualities are independent of the perception threshold $\tau$. That means, independent of how attentive consumers are, firms always choose the same qualities and the same prices. The attentiveness of consumers thus has no effect on producer surplus, consumer surplus, and welfare (see Proposition 4).

**Proposition 4** *When goods are not horizontally differentiated, neither producer surplus, consumer surplus, nor welfare depend on the perception threshold $\tau$.*

# 6 Extension: Marginal cost

To capture situations, where quality costs depend on the quantity that a firm sells, I extend the model in this section to marginal cost. I assume cost functions $C(q_i) = cq_ix_i$ with $c \in (0, 1)$, where $x_i$ is the demand for the good of firm $i$; all other marginal quantity costs are set to 0. I discuss two cases: In Section 6.1, I analyze the subgame-perfect equilibria when goods are horizontally differentiated. In Section 6.2, I analyze the subgame-perfect equilibria when goods are not horizontally differentiated.

## 6.1 Results with horizontal product differentiation

When the goods are horizontally differentiated and costs are given by $C(q_i) = cq_ix_i$ with $c \in (0, 1)$, the profit functions are[10]

$$\Pi_A(p_A, p_B, q_A, q_B) = (p_A - cq_A)\bar{x} \tag{4}$$

$$\Pi_B(p_A, p_B, q_A, q_B) = (p_B - cq_B)(1 - \bar{x}). \tag{5}$$

In the price-setting stage, firms simultaneously set prices to maximize profits given in (4) and (5) dependent on the qualities chosen in the first and second stage. As consumers' demand depends on the perceived qualities, the best replies also depend on the perceived

[10] If firms choose sufficiently different prices, the demand of one firm could become zero. However, in equilibrium, both firms choose prices such that both firms receive some demand.

qualities

$$p_A^*(p_B) = \frac{1 + p_B + cq_A + \hat{q}_A - \hat{q}_B}{2}$$
$$p_B^*(p_A) = \frac{1 + p_A + cq_B + \hat{q}_B - \hat{q}_A}{2}.$$

In equilibrium, firms thus choose prices

$$p_A^* = \frac{3 + 2cq_A + cq_B + \hat{q}_A - \hat{q}_B}{3}$$
$$p_B^* = \frac{3 + cq_A + 2cq_B + \hat{q}_B - \hat{q}_A}{3}.$$

The prices are higher compared to the fixed cost case, because firms account for the marginal costs in their prices. The equilibrium prices depend on the consumers' quality perceptions. That means, if the firms choose indistinguishable qualities, i.e., $q_B \in [q_A - \tau, q_A + \tau)$, the equilibrium prices are $p_A^* = (3 + 2cq_A + cq_B)/3$ and $p_B^* = (3 + 2cq_B + cq_A)/3$. When consumers are unable to notice the quality difference, they are unwilling to pay a quality premium. However, a firm's price is still increasing in its own quality: Higher quality implies higher marginal costs for producing the good, which has to be recovered through higher prices. In addition, a firm's price is increasing in the competitor's quality. If the competitor increases its quality, it has to raise the price, which allows the firm to raise its prices as well.

In contrast, if firms choose sufficiently different qualities such that consumers notice the quality difference, i.e., $q_B \in [0, q_A - \tau) \cap [q_A + \tau, 1]$, consumers perceive the qualities perfectly. Then, firms equilibrium prices are $p_A^* = (3 + 2cq_A + cq_B + q_A - q_B)/3$ and $p_B^* = (3 + 2cq_B + cq_A + q_B - q_A)/3$. When consumers are able to observe the quality difference, consumers are willing to pay a quality premium for goods with higher quality.

In the second stage, firm B chooses its quality $q_B$ to maximize its profit taking the prices into account. Firm B's profit is

$$\Pi_B(q_A, q_B) = \frac{1}{18}\Big(3 + c(q_A - q_B) + \hat{q}_B - \hat{q}_A\Big)^2. \tag{6}$$

The decision of firm B is driven by the following consideration: If firm B chooses a quality that is indistinguishable from the quality of firm A, i.e., $q_B \in [\max\{0, q_A - \tau\}, q_A + \tau)$, the consumers' demand does not depend directly on the qualities of the firms. The consumers' demand depends only indirectly on the qualities through the firms' prices. The lower the quality, the lower the price firm B can charge, which increases its demand. In addition, the lower the quality, the less cost firm B incurs. In sum, firm B's profit is decreasing in quality in the range $[\max\{0, q_A - \tau\}, q_A + \tau)$. Thus firm B prefers the lowest quality that is indistinguishable from the quality of firm A: $q_B = \max\{0, q_A - \tau\}$.

In contrast, if firm B chooses a distinguishable quality, i.e., $q_B \in [0, q_A - \tau) \cap [q_A + \tau, 1]$, the profit of firm B is increasing in its quality. If the quality difference is noticeable, the demand is directly affected by the quality. The higher the quality of a firm, the higher the demand. In addition, a higher quality allows firms to charge higher prices. Thus a higher quality increases the revenue of the firms which compensates the higher cost.

Whether the best reply of firm B is the highest noticeably different quality or the lowest unnoticeably different quality depends on the quality choice of firm A in the first stage. If firm A chooses a high quality, outperforming firm A noticeably is expensive or impossible. Thus firm B prefers the lowest quality that is indistinguishable from the quality of firm A. In contrast, if firm A chooses a low quality, firm B is able to produce goods with noticeably higher quality than firm A and thereby extract a quality premium from the consumers. Then, firm B chooses the highest quality that is distinguishable from the quality of firm A, $q_B = 1$. See Appendix D for a full characterization of the best replies of firm B.

In the first stage, firm A chooses its quality $q_A$ to maximize its profit taking the prices and the best reply of firm B into account. The profit of firm A depends on whether firm B responds with an unnoticeably different or a noticeably different quality

$$\Pi_A(q_A, q_B(q_A)) = \frac{1}{18}\Big(3 + cq_B^*(q_A) - cq_A + \hat{q}_A - \hat{q}_B^*(q_A)\Big)^2.$$

If firm A chooses a sufficiently low quality such that firm B responds with $q_B^*(q_A) = 1$, all consumers notice the quality difference. Then, firm A as the firm with the low-quality good makes less profit than firm B. Nevertheless, firm A has an incentive to choose the highest quality that induces firm B to respond with $q_B^*(q_A) = 1$: The higher its quality the more consumers are willing to pay. In contrast, if firm A chooses a sufficiently high quality such that firm B responds with $q_B^*(q_A) = \max\{0, q_A - \tau\}$, firm A has cost for quality but consumers do not notice the quality difference and are not willing to pay a quality premium for the good of firm A.

For all $\tau \in (0, 1)$ and for all $c \in (0, 1)$, firm A has an incentive to choose its quality $q_A$ such that firm B replies by unnoticeably undercutting firm A. However, for $\tau \in (0, c]$ with $c \in [1/2, 1)$, firm A is indifferent between choosing its quality $q_A$ such that firm B replies by unnoticeably undercutting firm A and by choosing its quality $q_A$ such that firm B replies with a noticeably higher quality. Thus for $\tau \in (0, c]$ with $c \in [1/2, 1)$, equilibria with indistinguishable qualities as well as an equilibrium with distinguishable qualities exist.

See Appendix D for a full derivation of the subgame-perfect equilibrium qualities. Proposition 5 summarizes the existence conditions for the two types of subgame-perfect equilibria.

**Proposition 5** *Subgame-perfect equilibria when firms have costs $C(q_i) = cq_ix_i$ with $c \in (0,1)$ and goods are horizontally differentiated:*

*(1)* ***Equilibrium with distinguishable qualities:*** *For $\tau \in (0,c]$ with $c \in [1/2,1)$, there exists a subgame-perfect equilibrium in which firms produce goods with distinguishable qualities.*

*(2)* ***Equilibrium with indistinguishable qualities:*** *For all $\tau \in (0,1)$ and all $c \in (0,1)$, there always exists at least one subgame-perfect equilibrium in which firms produce goods with indistinguishable qualities.*

If $c < 1/2$, in the subgame-perfect equilibria, firms always produce goods with indistinguishable qualities. Firm A always produces a higher quality than firm B. For $c \in [1/2,1)$, the subgame-perfect equilibria depend on $\tau$. For $\tau \in (0,1-c]$ a range of equilibria with indistinguishable qualities exists in addition to an equilibrium with distinguishable qualities. In the equilibria with indistinguishable qualities, firm A produces goods with a higher quality than firm B. In the equilibria with distinguishable qualities, firm B produces the higher quality. For $\tau \in (1-c,c]$, two equilibria exist: one equilibrium with indistinguishable qualities and one equilibrium with distinguishable qualities. For $\tau \in (c,1)$, a unique equilibrium with indistinguishable qualities exists.

Compared to the fixed cost case, this marginal cost case exhibits some differences, but captures the main results. Similarly to the fixed cost case, when consumers are not fully attentive to quality differences, an equilibrium exists where firms produce goods with indistinguishable qualities. In addition, qualities are weakly decreasing (within an equilibrium). Nevertheless, results also differ. With marginal costs, multiple equilibria with indistinguishable qualities can exist and even equilibria with distinguishable qualities can be sustained. Whereas with fixed cost, the equilibrium is unique.

## 6.2 Results without horizontal product differentiation

Without horizontal product differentiation, there always exists an equilibrium in which firms produce goods with distinguishable qualities as well as an equilibrium in which firms produce goods with indistinguishable qualities. If firms produce goods with indistinguishable qualities, price competition leads to prices equal to the marginal cost of the firm with the higher quality. Then, the firm with the higher quality makes zero profit and the firm with the lower quality makes positive profit. If firms produce goods with distinguishable qualities in the marginal cost case, price competition leads to prices such that the firm with the lower quality charges a price equal to its marginal cost and makes zero profit, whereas, the firm with the higher quality is able to charge a higher price that incorporates the quality difference and thus makes positive profits. Therefore, whenever firm A chooses a high quality, firm B as the second mover chooses the lowest quality that is indistinguishable from the quality of firm A. Whenever firm A chooses a low quality, firm B responds with a noticeably higher quality. In both cases firm A makes zero profit. Thus firm A is indifferent between the qualities, as they all yield the same profit. Therefore, there always exists an equilibrium in which firms produce goods with distinguishable qualities as well as an equilibrium in which firms produce goods with indistinguishable qualities.

Proposition 6 summarizes the results. See Appendix E for a detailed analysis.

**Proposition 6** *Subgame-perfect equilibria when firms have costs* $C(q_i) = cq_i x_i$ *with* $c \in (0,1)$ *and goods are not horizontally differentiated:*

*(1)* ***Equilibrium with distinguishable qualities:***
*For all* $\tau \in (0,1)$ *and all* $c \in (0,1)$*, there exists a subgame-perfect equilibrium in which firms produce goods with distinguishable qualities.*

*(2)* ***Equilibrium with indistinguishable qualities:***
*For all* $\tau \in (0,1)$ *and all* $c \in (0,1)$*, there exists a subgame-perfect equilibrium in which firms produce goods with indistinguishable qualities.*

Comparing the model with horizontal product differentiation and the model without horizontal product differentiation shows marked differences. The horizontal product differentiation dampens the price competition between the firms. In the marginal cost case, this has a limited effect. With and without horizontal product differentiation, there always exists an equilibrium with indistinguishable qualities. However, without horizontal product differentiation the equilibrium with distinguishable qualities exists for a larger range of values.

The lack of horizontal differentiation affects the equilibria in the marginal and in the fixed cost case differently. This difference hinges on the consequences of price competition. In the fixed cost case, both firms have the same marginal cost of 0. Then, if firms produce goods with indistinguishable qualities, price competition drives prices down to marginal cost and both firms make zero revenue. This makes producing indistinguishable qualities unattractive in the fixed cost case. In contrast, in the marginal cost case, firms may differ in their marginal cost. For firm A the marginal cost are $cq_A$ and for firm B the marginal cost are $cq_B$. Then, if firms produce goods with indistinguishable qualities, price competition drives prices down to the marginal cost of the firm with the higher marginal cost. Therefore, the firm with the lower marginal cost sets prices above its marginal cost and still makes positive profit. Thus in the fixed cost case, producing indistinguishable qualities is less attractive than in the marginal cost case.

# 7 Conclusion

In this article, I analyze the implications of consumers' limited attention to quality differences on equilibria and welfare. I capture this limited attention with a perception threshold such that consumers do not perceive quality differences between goods that are below their perception threshold. I show that in how far the perception threshold affects investments in quality depends on whether goods are horizontally differentiated as well as on the costs for quality.

When costs for quality are independent of the sold quantity, horizontal product differentiation affects the firms' incentives. With horizontal product differentiation, firms produce indistinguishable qualities. Without horizontal product differentiation, firms produce distinguishable qualities.

In contrast, when costs for quality are dependent on the sold quantity, horizontal product differentiation has a smaller effect. With and without horizontal product differentiation, an equilibrium with indistinguishable qualities always exists. However, this equilibrium need not be unique: A equilibrium with distinguishable qualities exists for some levels of inattention. When goods are not horizontally differentiated, the range of inattention, for which an equilibrium with distinguishable qualities exists, expands.

Understanding under which conditions firms produce goods with distinguishable and under which conditions firms produce goods with indistinguishable qualities is helpful for public policy. When firms produce goods with distinguishable qualities, consumers are fully informed and always choose the utility-maximizing good. In these situations, policy interventions to protect consumers are unnecessary. However, in many situations, firms produce goods with different, but indistinguishable qualities. When firms produce goods with different, but indistinguishable qualities, some consumers buy goods with lower than expected quality. Policy interventions should focus on these situations. Possible policy interventions include introducing labels that highlight the quality differences between the goods or sensitizing consumers to check the qualities more carefully in those markets.

# A Proof of Proposition 1

The proof proceeds in two parts. In the first part, I derive the best reply of firm B. In the second part, I derive the quality of firm A.

## Part 1: Best reply of firm B

In the second stage, firm B chooses its quality given the quality of firm A to maximize its profit given by

$$\begin{aligned}\Pi_B(q_A, q_B) =& \frac{1}{2}\left(1+\frac{\hat{q}_B-\hat{q}_A}{3}\right)^2-\frac{1}{2}q_B^2\\ =& \begin{cases}\frac{1}{2}\left(1+\frac{q_B-q_A}{3}\right)^2-\frac{1}{2}q_B^2 & \text{if } q_B < q_A-\tau\\ \frac{1}{2}-\frac{1}{2}q_B^2 & \text{if } q_A-\tau \leq q_B < q_A+\tau\\ \frac{1}{2}\left(1+\frac{q_B-q_A}{3}\right)^2-\frac{1}{2}q_B^2 & \text{if } q_A+\tau \leq q_A.\end{cases}\end{aligned} \tag{7}$$

Depending on $\tau$ and $q_A$ not all cases of (7) exist. The profit of firm B is strictly decreasing in $[\max\{0, q_A-\tau\}, q_A+\tau)$. Thus the profit in $[\max\{0, q_A-\tau\}, q_A+\tau)$ that yields the highest profit is $q_B = \max\{0, q_A-\tau\}$. In addition,

$$\frac{3-q_A}{8} = \arg\max_{q_B} \frac{1}{2}\left(1+\frac{q_B-q_A}{3}\right)^2-\frac{1}{2}q_B^2.$$

Although $0 \leq (3-q_A)/8 \leq 1$,

$$\begin{aligned}\frac{3-q_A}{8} &< q_A-\tau \Leftrightarrow q_A > \frac{3+8\tau}{9}\\ \frac{3-q_A}{8} &\geq q_A+\tau \Leftrightarrow q_A \leq \frac{3-8\tau}{9}.\end{aligned}$$

Thus the best reply of firm B is either $q_B = \max\{0, q_A-\tau\}$ or $q_B = (3-q_A)/8$ or a boundary solution. A comparison of the profits for the different candidates yields:

$$\begin{aligned}&\Pi_B\left(q_A, q_B = q_A-\tau\right) \geq \Pi_B\left(q_A, q_B = \frac{3-q_A}{8}\right)\\ \Leftrightarrow& \frac{3+8\tau}{9}-\frac{1}{9}\sqrt{48\tau-8\tau^2} \leq q_A \leq \frac{3+8\tau}{9}+\frac{1}{9}\sqrt{48\tau-8\tau^2}\\ &\Pi_B\left(q_A, q_B = 0\right) \geq \Pi_B\left(q_A, q_B = \frac{3-q_A}{8}\right) \Leftrightarrow q_A \geq 3-\sqrt{8}\\ &\Pi_B\left(q_A, q_B = 0\right) \geq \Pi_B\left(q_A, q_B = q_A+\tau\right) \Leftrightarrow q_A \geq -\tau+\frac{1}{3}\sqrt{6\tau+\tau^2}.\end{aligned}$$

In addition, if $(3-q_A)/8 \geq q_A-\tau$, $\Pi_B\left(q_A, q_B = q_A-\tau\right) > \Pi_B\left(q_A, q_B < q_A-\tau\right)$ and if $(3-q_A)/8 < q_A+\tau$, $\Pi_B\left(q_A, q_B = q_A-\tau\right) > \Pi_B\left(q_A, q_B = q_A+\tau\right)$.

Consequently:
(i) If $\tau < 3-\sqrt{8}$, the best reply of firm B is

$$q_B^*(q_A) = \begin{cases} \frac{3-q_A}{8} & \text{if } q_A \leq \frac{3+8\tau}{9} - \frac{1}{9}\sqrt{48\tau - 8\tau^2} \\ q_A - \tau & \text{if } \frac{3+8\tau}{9} - \frac{1}{9}\sqrt{48\tau - 8\tau^2} \leq q_A \leq \frac{3+8\tau}{9} + \frac{1}{9}\sqrt{48\tau - 8\tau^2} \\ \frac{3-q_A}{8} & \text{if } \frac{3+8\tau}{9} + \frac{1}{9}\sqrt{48\tau - 8\tau^2} \leq q_A. \end{cases}$$

(ii) If $3-\sqrt{8} \leq \tau \leq 3/4(3\sqrt{2}-4)$, the best reply of firm B is

$$q_B^*(q_A) = \begin{cases} \frac{3-q_A}{8} & \text{if } q_A \leq 3-\sqrt{8} \\ 0 & \text{if } 3-\sqrt{8} \leq q_A \leq \tau \\ q_A - \tau & \text{if } \tau < q_A \leq \frac{3+8\tau}{9} + \frac{1}{9}\sqrt{48\tau - 8\tau^2} \\ \frac{3-q_A}{8} & \text{if } \frac{3+8\tau}{9} + \frac{1}{9}\sqrt{48\tau - 8\tau^2} \leq q_A. \end{cases}$$

(iii) If $3/4(3\sqrt{2}-4) < \tau$, the best reply of firm B is

$$q_B^*(q_A) = \begin{cases} \frac{3-q_A}{8} & \text{if } q_A \leq \frac{3-8\tau}{9} \\ q_A + \tau & \text{if } \frac{3-8\tau}{9} < q_A \leq -\tau + \frac{1}{3}\sqrt{6\tau + \tau^2} \\ 0 & \text{if } -\tau + \frac{1}{3}\sqrt{6\tau + \tau^2} \leq q_A \leq \tau \\ q_A - \tau & \text{if } \tau < q_A \leq \frac{3+8\tau}{9} + \frac{1}{9}\sqrt{48\tau - 8\tau^2} \\ \frac{3-q_A}{8} & \text{if } \frac{3+8\tau}{9} + \frac{1}{9}\sqrt{48\tau - 8\tau^2} \leq q_A. \end{cases} \tag{8}$$

The last case of (8) only exists for $\tau \leq 1 - \sqrt{1/2}$, the first case exists for $\tau \leq 3/8$, and the second case exists for $\tau \leq 3/4$.

## Part 2: Quality of firm A

Generally, the profit of firm A is

$$\Pi_A(q_A, q_B) = \begin{cases} \frac{1}{2}\left(1 + \frac{q_A - q_B}{3}\right)^2 - \frac{1}{2}q_A^2 & \text{if } q_B < q_A - \tau \\ \frac{1}{2} - \frac{1}{2}q_A^2 & \text{if } q_A - \tau \leq q_B < q_A + \tau \\ \frac{1}{2}\left(1 + \frac{q_A - q_B}{3}\right)^2 - \frac{1}{2}q_A^2 & \text{if } q_A + \tau \leq q_B. \end{cases}$$

The profit of firm A depends on the response of firm B. In the first stage, firm A maximizes its profit taking the response of firm B into account. The following proof uses the fact that

$$\frac{21}{55} = \text{argmax}_{q_A} \frac{1}{2}\left(\frac{7+3q_A}{8}\right)^2 - \frac{1}{2}q_A^2$$

and that the corresponding profit is $\Pi_A(q_A = 21/55) = 49/110$. That means if the profit of firm A is $\Pi_A(q_A, q_B^*(q_A)) = 1/2\left((7+3q_A)/8\right)^2 - q_A^2/2$, then with any $q_A$, $\Pi_A(q_A, q_B^*(q_A)) \leq 49/110$.

**(i) Case $\tau < 3 - \sqrt{8}$:**

If $\tau < 3 - \sqrt{8}$, the best reply of firm B is

$$q_B^*(q_A) = \begin{cases} \frac{3-q_A}{8} & \text{if } q_A \leq \frac{3+8\tau}{9} - \frac{1}{9}\sqrt{48\tau - 8\tau^2} \\ q_A - \tau & \text{if } \frac{3+8\tau}{9} - \frac{1}{9}\sqrt{48\tau - 8\tau^2} \leq q_A \leq \frac{3+8\tau}{9} + \frac{1}{9}\sqrt{48\tau - 8\tau^2} \\ \frac{3-q_A}{8} & \text{if } \frac{3+8\tau}{9} + \frac{1}{9}\sqrt{48\tau - 8\tau^2} \leq q_A. \end{cases}$$

Then, the profit of firm A is

$$\Pi_A(q_A, q_B^*(q_A)) = \begin{cases} \frac{1}{2}\left(\frac{7+3q_A}{8}\right)^2 - \frac{1}{2}q_A^2 & \text{if } q_A \leq \frac{3+8\tau}{9} - \frac{1}{9}\sqrt{48\tau - 8\tau^2} \\ \frac{1}{2} - \frac{1}{2}q_A^2 & \text{if } \frac{3+8\tau}{9} - \frac{\sqrt{48\tau-8\tau^2}}{9} \leq q_A \leq \frac{3+8\tau}{9} + \frac{\sqrt{48\tau-8\tau^2}}{9} \\ \frac{1}{2}\left(\frac{7+3q_A}{8}\right)^2 - \frac{1}{2}q_A^2 & \text{if } \frac{3+8\tau}{9} + \frac{1}{9}\sqrt{48\tau - 8\tau^2} \leq q_A. \end{cases}$$

(1) For $q_A \leq (3+8\tau)/9 - \sqrt{48\tau - 8\tau^2}/9$, $21/55 = \text{argmax}_{q_A} \Pi_A(q_A, q_B^*(q_A))$ with $\Pi_A(q_A = 21/55) = 49/110$. Furthermore, $21/55 \in [0, (3+8\tau)/9 - \sqrt{48\tau - 8\tau^2}/9] \Leftrightarrow \tau \geq (21+\sqrt{433})/55$, which is never the case, because $\tau < 3 - \sqrt{8}$.

(2) For $(3+8\tau)/9 - \sqrt{48\tau - 8\tau^2}/9 \leq q_A \leq (3+8\tau)/9 + \sqrt{48\tau - 8\tau^2}/9$, $\partial\Pi_A(q_A, q_B^*(q_A))/\partial q_A < 0$, i.e., firm A makes the highest profit with $q_A = (3+8\tau)/9 - \sqrt{48\tau - 8\tau^2}/9$:
$\Pi_A(q_A = (3+8\tau)/9 - \sqrt{48\tau - 8\tau^2}/9) = 1/2 - 1/2\left((3+8\tau)/9 - \sqrt{48\tau - 8\tau^2}/9\right)^2$.

(3) For $(3+8\tau)/9 + \sqrt{48\tau - 8\tau^2}/9 \leq q_A$, the profit reaches its maximum at $q_A = 21/55$ with $\Pi_A(q_A = 21/55) = 49/110$. Furthermore, $21/55 \in [(3+8\tau)/9 + \sqrt{48\tau - 8\tau^2}/9, 1] \Leftrightarrow \tau \leq (21 - \sqrt{433})/55$.

Note that $\sqrt{6/55} - 1/2\sqrt{3\sqrt{6/55} - 61/110} < (21 - \sqrt{433})/55$. Thus for $\tau \leq \sqrt{6/55} - 1/2\sqrt{3\sqrt{6/55} - 61/110}$, $21/55 \in [(3+8\tau)/9 + \sqrt{48\tau - 8\tau^2}/9, 1]$. In addition,

$$\frac{49}{110} \leq \frac{1}{2} - \frac{1}{2}\left(\frac{3+8\tau}{9} - \frac{\sqrt{48\tau - 8\tau^2}}{9}\right)^2 \Leftrightarrow \tau \geq \sqrt{6/55} - 1/2\sqrt{3\sqrt{6/55} - 61/110}$$

Thus for $\tau \leq \sqrt{6/55} - 1/2\sqrt{3\sqrt{6/55} - 61/110}$, in the pure-strategy subgame-perfect equilibrium, firms choose

$$q_A^* = \frac{21}{55} \text{ and } q_B^* = \frac{18}{55}.$$

For $\sqrt{6/55} - 1/2\sqrt{3\sqrt{6/55} - 61/110} \leq \tau < 3 - \sqrt{8}$, in the pure-strategy subgame-perfect equilibrium, firms choose

$$q_A^* = \frac{3+8\tau}{9} - \frac{\sqrt{48\tau - 8\tau^2}}{9} \text{ and } q_B^* = \frac{3-\tau}{9} - \frac{\sqrt{48\tau - 8\tau^2}}{9}.$$

**(ii) Case $3 - \sqrt{8} \leq \tau \leq 3/4(3\sqrt{2} - 4)$:**

If $3 - \sqrt{8} \leq \tau \leq 3/4(3\sqrt{2} - 4)$, the best reply of firm B is

$$q_B^*(q_A) = \begin{cases} \frac{3-q_A}{8} & \text{if } q_A \leq 3 - \sqrt{8} \\ 0 & \text{if } 3 - \sqrt{8} \leq q_A \leq \tau \\ q_A - \tau & \text{if } \tau < q_A \leq \frac{3+8\tau}{9} + \frac{1}{9}\sqrt{48\tau - 8\tau^2} \\ \frac{3-q_A}{8} & \text{if } \frac{3+8\tau}{9} + \frac{1}{9}\sqrt{48\tau - 8\tau^2} \leq q_A. \end{cases}$$

Then, the profit of firm A is

$$\Pi_A(q_A, q_B^*(q_A)) = \begin{cases} \frac{1}{2}\left(\frac{7+3q_A}{8}\right)^2 - \frac{1}{2}q_A^2 & \text{if } q_A \leq 3 - \sqrt{8} \\ \frac{1}{2} - \frac{1}{2}q_A^2 & \text{if } 3 - \sqrt{8} \leq q_A \leq \frac{3+8\tau}{9} + \frac{1}{9}\sqrt{48\tau - 8\tau^2} \\ \frac{1}{2}\left(\frac{7+3q_A}{8}\right)^2 - \frac{1}{2}q_A^2 & \text{if } \frac{3+8\tau}{9} + \frac{1}{9}\sqrt{48\tau - 8\tau^2} \leq q_A. \end{cases}$$

(1) For $q_A \leq 3 - \sqrt{8}$, $\Pi_A(q_A, q_B^*(q_A)) \leq \Pi_A(q_A = 21/55) = 49/110$.

(2) For $3 - \sqrt{8} \leq q_A \leq (3+8\tau)/9 + \sqrt{48\tau - 8\tau^2}/9$, $\partial \Pi_A(q_A, q_B^*(q_A))/\partial q_A < 0$, i.e., firm A makes the highest profit with $q_A = 3 - \sqrt{8}$: $\Pi_A(q_A = 3 - \sqrt{8}) = 1/2 - 1/2\left(3 - \sqrt{8}\right)^2$.

(3) For $(3+8\tau)/9 + \sqrt{48\tau - 8\tau^2}/9 \leq q_A$, $\Pi_A(q_A, q_B^*(q_A)) \leq \Pi_A(q_A = 21/55) = 49/110$.

As $0 < 49/110 < 1/2 - 1/2\left(3 - \sqrt{8}\right)^2$, firm A chooses $q_A = 3 - \sqrt{8}$. Thus for $3 - \sqrt{8} \leq \tau \leq 3/4(3\sqrt{2} - 4)$, in the pure-strategy subgame-perfect equilibrium, firms choose

$$q_A^* = 3 - \sqrt{8} \text{ and } q_B^* = 0.$$

**(iii) Case $3/4(3\sqrt{2} - 4) < \tau$:**

If $3/4(3\sqrt{2} - 4) < \tau$, the best reply of firm B is

$$q_B^*(q_A) = \begin{cases} \frac{3-q_A}{8} & \text{if } q_A \leq \frac{3-8\tau}{9} \\ q_A + \tau & \text{if } \frac{3-8\tau}{9} < q_A \leq -\tau + \frac{1}{3}\sqrt{6\tau + \tau^2} \\ 0 & \text{if } -\tau + \frac{1}{3}\sqrt{6\tau + \tau^2} \leq q_A \leq \tau \\ q_A - \tau & \text{if } \tau < q_A \leq \frac{3+8\tau}{9} + \frac{1}{9}\sqrt{48\tau - 8\tau^2} \\ \frac{3-q_A}{8} & \text{if } \frac{3+8\tau}{9} + \frac{1}{9}\sqrt{48\tau - 8\tau^2} \leq q_A. \end{cases}$$

Then, the profit of firm A is

$$\Pi_A(q_A, q_B^*(q_A)) = \begin{cases} \frac{1}{2}\left(\frac{7+3q_A}{8}\right)^2 - \frac{1}{2}q_A^2 & \text{if } q_A \leq \frac{3-8\tau}{9} \\ \frac{1}{2}\left(1-\frac{\tau}{3}\right)^2 - \frac{1}{2}q_A^2 & \text{if } \frac{3-8\tau}{9} < q_A \leq -\tau + \frac{1}{3}\sqrt{6\tau+\tau^2} \\ \frac{1}{2} - \frac{1}{2}q_A^2 & \text{if } -\tau + \frac{1}{3}\sqrt{6\tau+\tau^2} \leq q_A \leq \frac{3+8\tau}{9} + \frac{\sqrt{48\tau-8\tau^2}}{9} \\ \frac{1}{2}\left(\frac{7+3q_A}{8}\right)^2 - \frac{1}{2}q_A^2 & \text{if } \frac{3+8\tau}{9} + \frac{1}{9}\sqrt{48\tau-8\tau^2} \leq q_A. \end{cases} \tag{9}$$

The last case of (9) only exists for $\tau \leq 1 - \sqrt{1/2}$, the first case exists for $\tau \leq 3/8$, and the second case exists for $\tau \leq 3/4$.

(1) For $q_A \leq (3-8\tau)/9$, $\Pi_A(q_A, q_B^*(q_A)) \leq \Pi_A(q_A = 21/55) = 49/110$.

(2) For $(3-8\tau)/9 < q_A \leq -\tau + 1/3\sqrt{6\tau+\tau^2}$, the profit of firm A is $\Pi_A(q_A, q_B^*(q_A)) = 1/2\,(1-\tau/3)^2 - 1/2q_A^2$.

(3) For $-\tau+1/3\sqrt{6\tau+\tau^2} \leq q_A \leq (3+8\tau)/9+\sqrt{48\tau-8\tau^2}/9$, $\partial\Pi_A(q_A, q_B^*(q_A))/\partial q_A < 0$, i.e., firm A makes the highest profit with $q_A = -\tau + 1/3\sqrt{6\tau+\tau^2}$:
$\Pi_A(q_A = -\tau + 1/3\sqrt{6\tau+\tau^2}) = 1/2 - 1/2\left(-\tau + 1/3\sqrt{6\tau+\tau^2}\right)^2$.

(4) For $(3+8\tau)/9 + \sqrt{48\tau-8\tau^2}/9 \leq q_A \leq 1$, $\Pi_A(q_A, q_B^*(q_A)) \leq \Pi_A(q_A = 21/55) = 49/110$.

As

$$\frac{1}{2}\left(1-\frac{\tau}{3}\right)^2 - \frac{1}{2}q_A^2 < \frac{49}{110} \text{ and}$$
$$\frac{49}{110} < \frac{1}{2} - \frac{1}{2}\left(-\tau + \frac{1}{3}\sqrt{6\tau+\tau^2}\right)^2,$$

firm A chooses $q_A = -\tau + 1/3\sqrt{6\tau+\tau^2}$. Thus for $3/4(3\sqrt{2}-4) < \tau < 3/4$, in the pure-strategy subgame-perfect equilibrium, firms choose

$$q_A^* = -\tau + \frac{1}{3}\sqrt{6\tau+\tau^2} \text{ and } q_B^* = 0$$

But as $-\tau + 1/3\sqrt{6\tau+\tau^2} \leq 0$ for $\tau \geq 3/4$, for $\tau \geq 3/4$:

$$q_A^* = q_B^* = 0.$$

## B Proof of Proposition 2

i) If $0 \leq \tau \leq \bar{\tau}$, in the subgame-perfect equilibrium, firms choose $q_A^* = 21/55$ and $q_B^* = 18/55$ and $p_A^* = 56/55$ and $p_B^* = 54/55$. Then, the indifferent consumer is $\bar{x} = 28/55$. The producer surplus, consumer surplus, and welfare are

$$PS = \Pi_A + \Pi_B = \frac{5287}{6050}$$
$$CS = \int_0^{\bar{x}} v + q_A^* - p_A^* - x^2 dx + \int_{\bar{x}}^1 v + q_B^* - p_B^* - (1-x)^2 dx = v - \frac{6613}{9075}$$
$$W = PS + CS.$$

ii) If $\bar{\tau} \leq \tau < 3 - \sqrt{8}$, in the subgame-perfect equilibrium, firms choose $q_A^* = (3+8\tau)/9 - \sqrt{48\tau - 8\tau^2}/9$ and $q_B^* = (3-\tau)/9 - \sqrt{48\tau - 8\tau^2}/9$ and $p_A^* = p_B^* = 1$. Then, the indifferent consumer is $\bar{x} = 1/2$. The producer surplus, consumer surplus, and welfare are

$$PS = \Pi_A + \Pi_B = 1 - \frac{1}{2}\left(q_A^*\right)^2 - \frac{1}{2}\left(q_B^*\right)^2$$
$$CS = \int_0^{\bar{x}} v + q_A^* - p_A^* - x^2 dx + \int_{\bar{x}}^1 v + q_B^* - p_B^* - (1-x)^2 dx$$
$$= v - \frac{3}{4} + \frac{7\tau}{18} - \frac{\sqrt{48\tau - 8\tau^2}}{9}$$
$$W = PS + CS.$$

iii) If $3 - \sqrt{8} \leq \tau \leq 3/4(3\sqrt{2} - 4)$, in the subgame-perfect equilibrium, firms choose $q_A^* = 3 - \sqrt{8}$ and $q_B^* = 0$ and $p_A^* = p_B^* = 1$. Then, the indifferent consumer is $\bar{x} = 1/2$. The producer surplus, consumer surplus, and welfare are

$$PS = \Pi_A + \Pi_B = 1 - \frac{1}{2}\left(q_A^*\right)^2$$
$$CS = \int_0^{\bar{x}} v + q_A^* - p_A^* - x^2 dx + \int_{\bar{x}}^1 v + q_B^* - p_B^* - (1-x)^2 dx = v + \frac{5}{12} - \sqrt{2}$$
$$W = PS + CS.$$

iv) If $3/4(3\sqrt{2} - 4) < \tau < 3/4$, in the subgame-perfect equilibrium, firms choose $q_A^* = -\tau + \sqrt{6\tau + \tau^2}/3$ and $q_B^* = 0$ and $p_A^* = p_B^* = 1$. Then, the indifferent consumer is $\bar{x} = 1/2$. The producer surplus, consumer surplus, and welfare are

$$PS = \Pi_A + \Pi_B = 1 - \frac{1}{2}\left(q_A^*\right)^2$$
$$CS = \int_0^{\bar{x}} v + q_A^* - p_A^* - x^2 dx + \int_{\bar{x}}^1 v + q_B^* - p_B^* - (1-x)^2 dx = v - \frac{13}{12} + \frac{1}{2}q_A^*$$
$$W = PS + CS.$$

v) If $3/4 \leq \tau$, in the subgame-perfect equilibrium, firms choose $q_A^* = q_B^* = 0$ and $p_A^* = p_B^* = 1$. Then, the indifferent consumer is $\bar{x} = 1/2$. The producer surplus, consumer surplus, and welfare are

$$\begin{aligned}
PS =& \Pi_A + \Pi_B = 1 \\
CS =& \int_0^{\bar{x}} v + q_A^* - p_A^* - x^2 dx + \int_{\bar{x}}^1 v + q_B^* - p_B^* - (1-x)^2 dx = v - \frac{13}{12} \\
W =& PS + CS.
\end{aligned}$$

**Producer Surplus:** For $\tau > \bar{\tau}$, each firm's revenue is $1/2$; the joint revenue is 1. In addition, for $\bar{\tau} < \tau < 3/4$, the firms have costs but for $\tau \geq 3/4$ the firms have no costs. Then, firms reach the highest producer surplus ($PS = 1$) in the range $\tau > \bar{\tau}$ for $\tau \geq 3/4$. In addition, as $1 > 5289/6050$, firms reach the overall highest producer surplus for $\tau \geq 3/4$.

**Consumer Surplus:** For $\tau > \bar{\tau}$, the firms qualities are indistinguishable and the firms sell the goods at the same price so that all consumers $x \leq \bar{x} = 1/2$ buy from firm A and all $x > \bar{x} = 1/2$ buy from firm B. Thus for $\tau > \bar{\tau}$, the highest consumer surplus is achieved when firms produce the goods with the highest quality. As for $\tau > \bar{\tau}$ firms (weakly) reduce their qualities with increasing $\tau$, firms produce the highest qualities in the range $\bar{\tau} < \tau < 3 - \sqrt{8}$. In addition,

$$v - \frac{6613}{9075} > v - \underbrace{\frac{3}{4}}_{> \frac{6613}{9075}} + \underbrace{\frac{7}{18}\tau - \frac{1}{9}\sqrt{48\tau - 8\tau^2}}_{<0}.$$

Thus the consumer surplus reaches its highest value for $\tau \leq \bar{\tau}$.

**Welfare:** For $\tau > \bar{\tau}$, the demand is fixed; each firm serves half the market. As prices are a reallocation of welfare from consumers to firms, welfare depends on the qualities. Consumers benefit from higher qualities, firms are harmed by higher qualities. With increasing $\tau$, qualities are weakly decreasing. In sum, welfare is weakly decreasing in $\tau$ such that the highest welfare for $\tau > \bar{\tau}$ is in the range $\bar{\tau} < \tau < 3 - \sqrt{8}$. Yet,

$$\begin{aligned}
1 - \frac{1}{2}\left(\frac{3+8\tau}{9} - \frac{1}{9}\sqrt{48\tau - 8\tau^2}\right)^2 - \frac{1}{2}\left(\frac{3-\tau}{9} - \frac{1}{9}\sqrt{48\tau - 8\tau^2}\right)^2 \\
+ v - \frac{3}{4} + \frac{7\tau}{18} - \frac{\sqrt{48\tau - 8\tau^2}}{9} < \frac{5287}{6050} + v - \frac{6613}{9075}.
\end{aligned}$$

Thus welfare reaches its highest value for $\tau \leq \bar{\tau}$.

## C Fixed costs without horizontal product differentiation

In the price-setting stage, the profits depend on the qualities chosen in the first and second stage. In the subgames with indistinguishable qualities, firm $i \in \{A, B\}$ with $i \neq j$ and $j \in \{A, B\}$ chooses price $p_i$ to maximize the following profit

$$\Pi_i(p_i, p_j, q_i, q_j) = \begin{cases} p_i - \frac{1}{2}q_i^2 & \text{if } p_i < p_j \\ \frac{1}{2}p_i - \frac{1}{2}q_i^2 & \text{if } p_i = p_j \\ 0 - \frac{1}{2}q_i^2 & \text{if } p_i > p_j. \end{cases}$$

This typical price competition yields prices equal to marginal cost $p_i^* = 0$ and zero revenue such that the profit is $\Pi_i(q_i, q_j) = -\frac{1}{2}q_i^2$.

In the subgames with distinguishable qualities, firm $i$ chooses price $p_i$ to maximize the following profit

$$\Pi_i(p_i, p_j, q_i, q_j) = \begin{cases} p_i - \frac{1}{2}q_i^2 & \text{if } p_i < p_j + q_i - q_j \\ \beta p_i - \frac{1}{2}q_i^2 & \text{if } p_i = p_j + q_i - q_j \\ 0 - \frac{1}{2}q_i^2 & \text{if } p_i > p_j + q_i - q_j, \end{cases}$$

where $\beta = 0$ if $q_i < q_j$, $\beta = 1/2$ if $q_i = q_j$, and $\beta = 1$ if $q_i > q_j$. Assume $q_i > q_j$. The intense price competition ensures that, in equilibrium, the prices depend on the qualities of the firms

$$p_i^* = q_i - q_j \text{ and } p_j^* = 0.$$

The corresponding profits are

$$\Pi_i(q_i, q_j) = q_i - q_j - \frac{1}{2}q_i^2 \text{ and } \Pi_j(q_i, q_j) = -\frac{1}{2}q_j^2.$$

In the second stage, taking the results from the price-setting stage into account, firm B chooses its quality $q_B$ to maximize the following profit

$$\Pi_B(q_A, q_B) = \begin{cases} -\frac{1}{2}q_B^2 & \text{if } q_B < q_A - \tau \\ -\frac{1}{2}q_B^2 & \text{if } q_A - \tau \leq q_B < q_A + \tau \\ q_B - q_A - \frac{1}{2}q_B^2 & \text{if } q_A + \tau \leq q_B. \end{cases} \tag{10}$$

If $q_A \leq \tau$, the first case of (10) does not exist. If $q_A > 1 - \tau$, the third case of (10) does not exist. If firm B chooses a quality $q_B \in [0, q_A + \tau)$ the profit of firm B is

$\Pi_B(q_A, q_B) = -1/2q_B^2$. Thus the quality in $q_B \in [0, q_A + \tau)$ that yields the highest profit for firm B is $q_B = 0$ with $\Pi_B(q_A, q_B = 0) = 0$. If firm B chooses a quality $q_B \in [q_A + \tau, 1]$, the profit of firm B is $\Pi_B(q_A, q_B) = q_B - q_A - 1/2q_B^2$. Thus the quality in $q_B \in [q_A + \tau, 1]$ that yields the highest profit for firm B is $q_B = 1$ with $\Pi_B(q_A, q_B = 1) = 1/2 - q_A$.

The best reply of firm B depends on $c$ and $\tau$. If $\tau < 1/2$, the best reply of firm B is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } q_A \leq \frac{1}{2} \\ 0 & \text{if } q_A \geq \frac{1}{2}. \end{cases} \tag{11}$$

Firm B always chooses a quality that is noticeably different from the quality of firm A.

If $\tau = 1/2$, the best reply of firm B is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } q_A \leq \frac{1}{2} \\ 0 & \text{if } q_A = \frac{1}{2} \\ 0 & \text{if } q_A > \frac{1}{2}. \end{cases} \tag{12}$$

If firm A chooses quality $q_A = 1/2$, firm B's best reply is to choose a quality that is unnoticeably different, otherwise firm B chooses a quality that is noticeably different.

If $\tau > 1/2$, the best reply of firm B is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } q_A \leq 1 - \tau \\ 0 & \text{if } 1 - \tau < q_A \leq \tau \\ 0 & \text{if } q_A > \tau. \end{cases} \tag{13}$$

If firm A chooses a quality $q_A \in (1 - \tau, \tau]$, firm B's best reply is to choose a quality that is unnoticeably different, otherwise firm B chooses a quality that is noticeably different.

In the first stage, firm A takes the best reply of firm B into account, when it chooses its quality. If $\tau < 1/2$, the best reply of firm B is given in (11) and the profit of firm A is

$$\Pi_A(q_A, q_B^*(q_A)) = \begin{cases} -\frac{1}{2}q_A^2 & \text{if } q_A \leq \frac{1}{2} \\ q_A - \frac{1}{2}q_A^2 & \text{if } q_A \geq \frac{1}{2}. \end{cases}$$

As $\partial\Pi(q_A, q_B^*(q_A))/\partial q_A < 0$ for $q_A \leq 1/2$ and $\partial\Pi(q_A, q_B^*(q_A))/\partial q_A > 0$ for $q_A \geq 1/2$, firm A either chooses $q_A = 0$ with profit $\Pi(q_A = 0, q_B^*(q_A)) = 0$ or $q_A = 1$ with profit $\Pi(q_A = 1, q_B^*(q_A)) = 1/2$. As $1/2 > 0$, firm A chooses $q_A^* = 1$.

If $\tau = 1/2$, the best reply of firm B is given in (12) and the profit of firm A is

$$\Pi_A(q_A, q_B^*(q_A)) = \begin{cases} -\frac{1}{2}q_A^2 & \text{if } q_A \leq \frac{1}{2} \\ -\frac{1}{2}q_A^2 & \text{if } q_A = \frac{1}{2} \\ q_A - \frac{1}{2}q_A^2 & \text{if } q_A > \frac{1}{2}. \end{cases}$$

As $\partial\Pi(q_A, q_B^*(q_A))/\partial q_A < 0$ for $q_A \leq 1/2$ and $\partial\Pi(q_A, q_B^*(q_A))/\partial q_A > 0$ for $q_A > 1/2$, firm A either chooses $q_A = 0$ with profit $\Pi(q_A = 0, q_B^*(q_A)) = 0$ or $q_A = 1$ with profit $\Pi(q_A = 1, q_B^*(q_A)) = 1/2$. As $1/2 > 0$, firm A chooses $q_A^* = 1$.

If $\tau > 1/2$, the best reply of firm B is given in (13) and the profit of firm A is

$$\Pi_A(q_A, q_B^*(q_A)) = \begin{cases} -\frac{1}{2}q_A^2 & \text{if } q_A \leq 1-\tau \\ -\frac{1}{2}q_A^2 & \text{if } 1-\tau < q_A \leq \tau \\ q_A - \frac{1}{2}q_A^2 & \text{if } q_A > \tau. \end{cases}$$

As $\partial\Pi(q_A, q_B^*(q_A))/\partial q_A < 0$ for $q_A \leq \tau$ and $\partial\Pi(q_A, q_B^*(q_A))/\partial q_A > 0$ for $q_A > \tau$, firm A either chooses $q_A = 0$ with profit $\Pi(q_A = 0, q_B^*(q_A)) = 0$ or $q_A = 1$ with profit $\Pi(q_A = 1, q_B^*(q_A)) = 1/2$. As $1/2 > 0$, firm A chooses $q_A^* = 1$.

Consequently, for all $\tau \in (0, 1)$, there exists a subgame-perfect equilibrium in which firm A chooses $q_A^* = 1$ and firm B chooses $q_B^* = 0$ and the qualities are distinguishable. Firm A makes a profit of $\Pi_A = 1/2$ and firm B makes zero profit $\Pi_B = 0$. Thus with fixed costs but without horizontal product differentiation, firm A has a first-mover advantage.

# D Marginal costs with horizontal product differentiation

The profit of firm B is

$$\Pi_B(q_A, q_B) = \frac{1}{18}\Big(3 + c(q_A - q_B) + \hat{q}_B - \hat{q}_A\Big)^2$$
$$= \begin{cases} \frac{1}{18}\Big(3 + (1-c)(q_B - q_A)\Big)^2 & \text{if } q_B < q_A - \tau \\ \frac{1}{18}\Big(3 + c(q_A - q_B)\Big)^2 & \text{if } q_A - \tau \le q_B < q_A + \tau \\ \frac{1}{18}\Big(3 + (1-c)(q_B - q_A)\Big)^2 & \text{if } q_A + \tau \le q_B. \end{cases} \tag{14}$$

The profit is decreasing in the interval $[q_A - \tau, q_A + \tau)$ and increasing otherwise. However, if $q_A \le \tau$, the first case of (14) does not exist and, if $q_A > 1 - \tau$, the third case of (14) does not exist. In addition, $\Pi_B(q_A, q_B < q_A - \tau) < \Pi_B(q_A, q_B = q_A - \tau)$. Thus the best reply of firm B is either $q_B = \max\{0, q_A - \tau\}$ or $q_B = 1$.

## D.1 $\tau \le 1/2$

Assume $\tau \le 1/2$.

(1) If $c < 1/2$, the best reply of firm B is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } 0 \le q_A \le 1 - \tau \\ q_A - \tau & \text{if } 1 - \tau < q_A \le 1. \end{cases}$$

In the first stage, taking the best reply into account, firm A chooses its quality $q_A$ to maximize its profit

$$\Pi_A(q_A, q_B^*(q_A)) = \begin{cases} \frac{1}{18}(3 + q_A - 1 + c - cq_A)^2 & \text{if } 0 \le q_A \le 1 - \tau \\ \frac{1}{18}(3 - c\tau)^2 & \text{if } 1 - \tau < q_A \le 1. \end{cases}$$

As $1/18(3 + q_A - 1 + c - cq_A)^2 < 1/18(3 - c\tau)^2$ for all $q_A \in [0, 1 - \tau]$, any quality pair

$$(q_A^*, q_B^*) \in \{(q_A, q_B) | q_A \in (1 - \tau, 1], q_B \in (1 - 2\tau, 1 - \tau], q_B = q_A - \tau\}$$

with prices $p_A^* = 1/3(3 + 3cq_A^* - c\tau)$ and $p_B^* = 1/3(3 + 3cq_A^* - 2c\tau)$ constitutes a subgame-perfect equilibrium. The corresponding equilibrium profits are $\Pi_A^* = 1/18(3 - c\tau)^2$ and $\Pi_B^* = 1/18(3 + c\tau)^2$. In all subgame-perfect equilibria, the qualities are indistinguishable.

(2) If $c \geq 1/2$ and $\tau < 1-c$, the best reply of firm B is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } 0 \leq q_A \leq 1 - \frac{c\tau}{1-c} \\ q_A - \tau & \text{if } 1 - \frac{c\tau}{1-c} \leq q_A \leq 1. \end{cases}$$

In the first stage, taking the best reply into account, firm A then chooses its quality $q_A$ to maximize its profit

$$\Pi_A(q_A, q_B^*(q_A)) = \begin{cases} \frac{1}{18}(3 + q_A - 1 + c - cq_A)^2 & \text{if } 0 \leq q_A \leq 1 - \frac{c\tau}{1-c} \\ \frac{1}{18}(3 - c\tau)^2 & \text{if } 1 - \frac{c\tau}{1-c} \leq q_A \leq 1. \end{cases}$$

As $1/18(3 + q_A - 1 + c - cq_A)^2 < 1/18(3 - c\tau)^2$ for all $q_A \in [0, 1 - c\tau/(1-c))$ and $1/18(3 + q_A - 1 + c - cq_A)^2 = 1/18(3 - c\tau)^2$ for $q_A = 1 - c\tau/(1-c)$, any quality pair

$$(q_A^*, q_B^*) \in \{(q_A, q_B) | q_A \in [1 - \frac{c\tau}{1-c}, 1], q_B \in [1 - \frac{\tau}{1-c}, 1 - \tau], q_B = q_A - \tau\}$$

with prices $p_A^* = 1/3(3 + 3cq_A^* - c\tau)$ and $p_B^* = 1/3(3 + 3cq_A^* - 2c\tau)$ constitutes a subgame-perfect equilibrium. The corresponding equilibrium profits are $\Pi_A^* = 1/18(3 - c\tau)^2$ and $\Pi_B^* = 1/18(3 + c\tau)^2$. In these subgame-perfect equilibria, the qualities are indistinguishable. In addition, there exists a subgame-perfect equilibrium, where

$$q_A^* = 1 - \frac{c\tau}{1-c} \text{ and } q_B^* = 1$$

with prices $p_A^* = 1/3(3 + 3c - c\tau(1 + 2c)/(1 - c))$ and $p_B^* = 1/3(3 + 3c + c\tau)$. The corresponding equilibrium profits are $\Pi_A^* = 1/18(3 - c\tau)^2$ and $\Pi_B^* = 1/18(3 + c\tau)^2$. In this equilibrium, the qualities are distinguishable.

(3) If $c \geq 1/2$ and $\tau \geq 1 - c$, the best reply of firm B is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } 0 \leq q_A \leq 1 - c \\ 0 & \text{if } 1 - c \leq q_A \leq \tau \\ q_A - \tau & \text{if } \tau < q_A \leq 1. \end{cases}$$

In the first stage, taking the best reply into account, firm A chooses its quality $q_A$ to maximize its profit

$$\Pi_A(q_A, q_B^*(q_A)) = \begin{cases} \frac{1}{18}(3 + q_A - 1 + c - cq_A)^2 & \text{if } 0 \leq q_A \leq 1 - c \\ \frac{1}{18}(3 - cq_A)^2 & \text{if } 1 - c \leq q_A \leq \tau \\ \frac{1}{18}(3 - c\tau)^2 & \text{if } \tau < q_A \leq 1. \end{cases}$$

The profit is increasing in $[0, 1-c)$ and decreasing in $(1-c, \tau]$. The profit with $q_A = 1-c$

is $\Pi_A = 1/18(3-c+c^2)^2$. As $1/18(3-c+c^2)^2 \geq 1/18(3-c\tau)^2$, there exists one subgame-perfect equilibrium with distinguishable qualities where

$$q_A^* = 1-c \text{ and } q_B^* = 1$$

with prices $p_A^* = 1/3(3+2c-2c^2)$ and $p_B^* = 1/3(3+4c-c^2)$ and one subgame-perfect equilibrium with indistinguishable qualities where

$$q_A^* = 1-c \text{ and } q_B^* = 0$$

with prices $p_A^* = 1/3(3+2c-2c^2)$ and $p_B^* = 1/3(3+c-c^2)$. The corresponding equilibrium profits for both subgame-perfect equilibria are $\Pi_A^* = 1/18(3-c+c^2)^2$ and $\Pi_B^* = 1/18(3+c-c^2)^2$.

In addition, for $\tau = 1-c$, any quality pair

$$(q_A^*, q_B^*) \in \{(q_A, q_B) | q_A \in (\tau, 1], q_B \in (0, 1-\tau], q_B = q_A - \tau\}$$

with prices $p_A^* = 1/3(3+3cq_A^*-c\tau)$ and $p_B^* = 1/3(3+3cq_A^*-2c\tau)$ constitutes a subgame-perfect equilibrium. The corresponding equilibrium profits are $\Pi_A^* = 1/18(3-c\tau)^2$ and $\Pi_B^* = 1/18(3+c\tau)^2$.

## D.2 $\tau > 1/2$

Assume $\tau > 1/2$.

(1) If $\tau \leq c$, the best reply of firm B is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } 0 \leq q_A \leq 1-c \\ 0 & \text{if } 1-c \leq q_A \leq \tau \\ q_A - \tau & \text{if } \tau < q_A \leq 1. \end{cases}$$

In the first stage, taking the best reply into account, firm A chooses its quality $q_A$ to maximize its profit

$$\Pi_A(q_A, q_B^*(q_A)) = \begin{cases} \frac{1}{18}(3+q_A-1+c-cq_A)^2 & \text{if } 0 \leq q_A \leq 1-c \\ \frac{1}{18}(3-cq_A)^2 & \text{if } 1-c \leq q_A \leq \tau \\ \frac{1}{18}(3-c\tau)^2 & \text{if } \tau < q_A \leq 1. \end{cases}$$

The profit is increasing in $[0, 1-c)$ and decreasing in $(1-c, \tau]$. The profit with $q_A = 1-c$ is $\Pi_A = 1/18(3-c+c^2)^2$, and $1/18(3-c+c^2)^2 \geq 1/18(3-c\tau)^2 \Leftrightarrow \tau \geq 1-c$. There exists one subgame-perfect equilibrium with distinguishable qualities where

$$q_A^* = 1 - c \text{ and } q_B^* = 1$$

with prices $p_A^* = 1/3(3 + 2c - 2c^2)$ and $p_B^* = 1/3(3 + 4c - c^2)$ and one subgame-perfect equilibrium with indistinguishable qualities where

$$q_A^* = 1 - c \text{ and } q_B^* = 0$$

with prices $p_A^* = 1/3(3+2c-2c^2)$ and $p_B^* = 1/3(3+c-c^2)$. The corresponding equilibrium profits for both subgame-perfect equilibria are $\Pi_A^* = 1/18(3-c+c^2)^2$ and $\Pi_B^* = 1/18(3+c-c^2)^2$.

(2) If $\tau > c$, the best reply of firm B is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } 0 \leq q_A \leq 1-\tau \\ 0 & \text{if } 1-\tau < q_A \leq \tau \\ q_A - \tau & \text{if } \tau < q_A \leq 1. \end{cases}$$

In the first stage, taking the best reply into account, firm A chooses its quality $q_A$ to maximize its profit

$$\Pi_A(q_A, q_B^*(q_A)) = \begin{cases} \frac{1}{18}(3 + q_A - 1 + c - cq_A)^2 & \text{if } 0 \leq q_A \leq 1-\tau \\ \frac{1}{18}(3 - cq_A)^2 & \text{if } 1-\tau < q_A \leq \tau \\ \frac{1}{18}(3 - c\tau)^2 & \text{if } \tau < q_A \leq 1. \end{cases}$$

The profit is increasing in $[0, 1-\tau]$ and decreasing in $(1-\tau, \tau]$. As $1/18(3-cq_A)^2 > 1/18(3-\tau+c\tau)^2 \Leftrightarrow q_A < \tau(1-c)/c$ with $\tau(1-c)/c > 1-\tau$ and as $1/18(3-cq_A)^2 \geq 1/18(3-c\tau)^2$ for all $q_A \in (1-\tau, \tau]$, firm A chooses the lowest quality in the interval $(1-\tau, \tau]$. Assume that there exists a smallest quality unit such that $\underline{q}_A$ is the lowest quality $q_A$ in $(1-\tau, \tau]$. Then, there exists one subgame-perfect equilibrium with indistinguishable qualities where

$$q_A^* = \underline{q}_A \text{ and } q_B^* = 0$$

with prices $p_A^* = 1/3(3 + 2c\underline{q}_A)$ and $p_B^* = 1/3(3 + c\underline{q}_A)$. The corresponding equilibrium profits are $\Pi_A^* = 1/18(3 - c\underline{q}_A)^2$ and $\Pi_B^* = 1/18(3 + c\underline{q}_A)^2$.

# E Marginal cost without horizontal product differentiation

Assume costs: $C_i(q_i) = cq_i x_i$ where $c \in (0,1)$. Without horizontal product differentiation, consumers' utility and perceived utility from buying the good of firm $i \in \{A, B\}$ are

$$u(i) = v + q_i - p_i$$
$$\hat{u}(i) = v + \hat{q}_i - p_i.$$

Consumers buy from firm $i \in \{A, B\}$ with $i \neq j$ and $j \in \{A, B\}$ if

$$\hat{u}(i) > \hat{u}(j) \Leftrightarrow p_i < \hat{q}_i - \hat{q}_j + p_j.$$

To sustain the equilibrium, I make the following assumptions about consumers' behavior when they are indifferent between the two firms, i.e., when $\hat{u}(i) = \hat{u}(j)$. First, consumers buy from the firm with the higher quality if the qualities are distinguishable. Second, consumers buy from the firm with the lower quality if the qualities are indistinguishable.[11]

I solve the game by backward induction, starting with the price-setting stage. In the subgames with indistinguishable qualities, firm $i \in \{A, B\}$ with $i \neq j$ and $j \in \{A, B\}$ chooses price $p_i$ to maximize the following profit

$$\Pi_i(p_i, p_j, q_i, q_j) = \begin{cases} p_i - cq_i & \text{if } p_i < p_j \\ \alpha(p_i - cq_i) & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j, \end{cases}$$

where $\alpha = 0$ if $q_i > q_j$, $\alpha = 1/2$ if $q_i = q_j$, and $\alpha = 1$ if $q_i < q_j$. That means, without horizontal product differentiation, the intense price competition leads to prices equal to the marginal cost of the firm with the higher quality: In equilibrium, firm $i$'s price is $p_i^* = \max\{cq_i, cq_j\}$ and its profit is $\Pi_i(q_i, q_j) = p_i^* - cq_i$.

In the subgames with distinguishable qualities, firm $i$ chooses price $p_i$ to maximize the following profit

$$\Pi_i(p_i, p_j, q_i, q_j) = \begin{cases} p_i - cq_i & \text{if } p_i < p_j + q_i - q_j \\ \beta(p_i - cq_i) & \text{if } p_i = p_j + q_i - q_j \\ 0 & \text{if } p_i > p_j + q_i - q_j, \end{cases}$$

[11] These assumptions simply sustain the equilibria. Very similar equilibria occur without this assumption, because the firm that receives the complete demand is always able to set a marginally lower price to capture the complete demand.

where $\beta = 0$ if $q_i < q_j$, $\beta = 1/2$ if $q_i = q_j$, and $\beta = 1$ if $q_i > q_j$. The intense price competition ensures that the firm with the lower quality chooses prices equal to marginal cost and makes zero profit. However, the firm with the higher quality now benefits from the quality difference between the goods. In equilibrium, the prices depend on the qualities of the firms. Assume $q_i > q_j$, then the equilibrium prices are

$$p_i^* = q_i - q_j + cq_j$$
$$p_j^* = cq_j.$$

The corresponding profits are[12]

$$\Pi_i(q_i, q_j) = (q_i - q_j)(1 - c)$$
$$\Pi_j(q_i, q_j) = 0.$$

In the second stage, taking the results from the price-setting stage into account, firm B chooses its quality $q_B$ to maximize the following updated profit

$$\Pi_B(q_A, q_B) = \begin{cases} 0 & \text{if } q_B < q_A - \tau \\ \max\{cq_A, cq_B\} - cq_B & \text{if } q_A - \tau \leq q_B < q_A + \tau \\ (q_B - q_A)(1 - c) & \text{if } q_A + \tau \leq q_B. \end{cases} \tag{15}$$

If $q_A \leq \tau$, the first case of (15) does not exist. And if $q_A > 1 - \tau$, the third case of (15) does not exist. If firm B chooses a quality $q_B \in [0, q_A - \tau)$ the profit of firm B is $\Pi_B(q_A, q_B) = 0$. Thus all $q_B \in [0, q_A - \tau)$ yield the same profit. If firm B chooses a quality $q_B \in [q_A - \tau, q_A + \tau)$, the profit of firm B is $\Pi_B(q_A, q_B) = \max\{cq_A, cq_B\} - cq_B$. Thus the quality $q_B \in [q_A - \tau, q_A + \tau)$ that yields the highest profit is $q_B = \max\{0, q_A - \tau\}$ with profit $\Pi_B(q_A, q_B = 0) = cq_A$ if $q_A \leq \tau$ and $\Pi_B(q_A, q_B = q_A - \tau) = c\tau$ if $q_A > \tau$. If firm B chooses a quality $q_B \in [q_A + \tau, 1]$, the profit of firm B is $\Pi_B(q_A, q_B) = (q_B - q_A)(1 - c)$. Thus the quality $q_B \in [q_A + \tau, 1]$ that yields the highest profit is $q_B = 1$ with profit $\Pi_B(q_A, q_B = 1) = (1 - q_A)(1 - c) > 0$.

Consequently, the best reply of firm B is either $q_B(q_A) = 1$ (distinguishable), $q_B(q_A) = 0$ (indistinguishable), or $q_B(q_A) = q_A - \tau$ (indistinguishable). As

$$cq_A \geq (1 - q_A)(1 - c) \Leftrightarrow q_A \geq 1 - c$$
$$c\tau \geq (1 - q_A)(1 - c) \Leftrightarrow q_A \geq 1 - \frac{c\tau}{1 - c},$$

the best reply of firm B depends on $c$ and $\tau$: If $\tau \leq 1/2$, $\tau < 1 - c$, and $c \geq 1/2$, the best

[12]All consumers buy from the firm with the higher quality, otherwise this firm could decrease the price marginally to capture all consumers.

reply of firm B is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } q_A \le 1 - \frac{c\tau}{1-c} \\ q_A - \tau & \text{if } q_A \ge 1 - \frac{c\tau}{1-c}. \end{cases}$$

If $\tau \le 1/2$, $\tau \ge 1 - c$, and $c \ge 1/2$, or if $\tau > 1/2$ and $\tau \le c$, the best reply of firm B is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } q_A \le 1 - c \\ 0 & \text{if } 1 - c \le q_A \le \tau \\ q_A - \tau & \text{if } q_A > \tau. \end{cases}$$

If $\tau \le 1/2$ and $c < 1/2$, the best reply of firm B is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } q_A \le 1 - \tau \\ q_A - \tau & \text{if } q_A > 1 - \tau. \end{cases}$$

If $\tau > 1/2$ and $\tau > c$, the best reply of firm B is

$$q_B^*(q_A) = \begin{cases} 1 & \text{if } q_A \le 1 - \tau \\ 0 & \text{if } 1 - \tau < q_A \le \tau \\ q_A - \tau & \text{if } q_A > \tau. \end{cases}$$

In the first stage, firm A chooses its quality $q_A$ to maximize its profit taking the best reply of firm B into account. However, with all $q_A \in [0, 1]$, firm A makes profit $\Pi_A(q_A, q_B^*(q_A)) = 0$. Thus firm A is indifferent between all qualities $q_A \in [0, 1]$. That means, this game has infinitely many subgame-perfect Nash equilibria. For any $\tau \in (0, 1)$ and $c \in (0, 1)$, there exist equilibria in which firm A chooses a quality that is sufficiently low such that firm B responds with $q_B^* = 1$ and the quality difference is noticeable. In addition, for any $\tau \in (0, 1)$ and $c \in (0, 1)$, there exist equilibria in which firm A chooses a high quality and firm B responds by undercutting this quality unnoticeably: $q_B^* = \max\{0, q_A - \tau\}$. The profit of firm A is always $\Pi_A = 0$. The profit of firm B depends on the subgame-perfect equilibirum: In equilibria with $q_B^* = 1$, firm B makes profit $\Pi_B = (1 - q_A^*)(1 - c)$. In equilibria with $q_B^* = 0$, firm B makes profit $\Pi_B = cq_A^*$. In equilibria with $q_B^* = q_A^* - \tau$, firm B makes profit $\Pi_B = c\tau$. In all equilibria, firm B makes a strictly positive profit. Consequently, with marginal costs but without horizontal product differentiation, firm B has a second-mover advantage.

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