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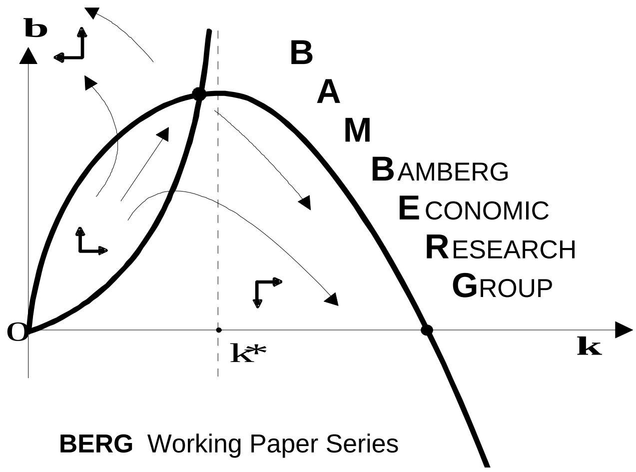

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Trend followers, contrarians and fundamentalists: explaining the dynamics of financial markets☆

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Abstract

We propose an empirically motivated financial market model in which speculators rely on trend-following, contrarian and fundamental trading rules to determine their orders. Speculators' probabilistic rule-selection behavior – the only type of randomness in our model – depends on past and future performance indicators. For a large number of speculators, the model's intrinsic noise vanishes and its dynamics is driven by an analytically tractable nonlinear map. An in-depth investigation into this map provides the key to understanding how the model functions. Since our model is able to match a number of important stylized facts concerning financial markets, it may be regarded as validated.

Keywords: Financial markets; stylized facts; technical and fundamental analysis; probabilistic rule-selection behavior; nonlinear dynamics; stability and bifurcation analysis.
JEL classification: C63; D84; G15.

1. Introduction

Shiller (2015) vividly argues that the instability of financial markets, resulting largely from the irrational exuberance of boundedly rational and heterogeneous agents, may be quite harmful to the real economy. Two of the most notorious examples in this respect include the worldwide stock market crash of 1929, which triggered the Great Depression, and the global financial turmoil of 2007, which led to the Great Recession. In the last couple of years, models involving heterogeneous interacting agents, addressed in Dieci and He (2018) and Iori and Porter (2018), have considerably improved our understanding of how financial markets function. Pioneering

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contributions by Beja and Goldman (1980), Day and Huang (1990), Kirman (1993), Lux (1995) and Brock and Hommes (1998) demonstrate that nonlinear interactions between chartists and fundamentalists may cause complex price dynamics. Some of these models are even capable of replicating the statistical properties of financial markets in finer detail. Powerful examples in this direction include LeBaron et al. (1999), Lux and Marchesi (1999), Farmer and Joshi (2002), de Grauwe and Grimaldi (2006) and Westerhoff and Dieci (2006). Unfortunately, these models are either rather complex, making it difficult to pinpoint the causalities acting within them, or rely heavily on exogenous noise, preventing a true endogenous explanation of how financial markets function.

From the very beginning, one of the key goals of this line of research has been to find a rather general yet analytically tractable model that endogenously explains the dynamics of financial markets. The sheer complexity of this task is one of the main reasons why this goal has as yet not been fully accomplished. One major initial challenge for such a model is to produce serially uncorrelated price changes – a crucial feature of actual financial markets. However, the ultimate quest is to develop an economically credible setup that mimics a whole battery of stylized facts of financial markets, comprising diverse properties such as bubbles and crashes, excess volatility, fat-tailed return distributions, serially uncorrelated price changes and volatility clustering. It is, of course, very difficult to achieve such a goal, which is why no such models have yet been found.

Our paper seeks to make progress in this direction. In fact, we propose a fairly general, analytically tractable and empirically motivated model that gives an account of the intricate dynamics of financial markets without relying on exogenous noise. With respect to our model setup, it is important to note that laboratory evidence (Hommes et al. 2005, Bao et al. 2017) and questionnaire studies (Taylor and Allen 1992, Menkhoff and Taylor 2007, Menkhoff 2010) indicate that speculators rely on technical and fundamental trading rules to determine their orders. Moreover, Anufriev and Hommes (2012) and Dick and Menkhoff (2013) report that speculators switch between trading rules according to past and future performance indicators. A similar conclusion can be drawn from Boswijk et al. (2007), Franke and Westerhoff (2012) and Chiarella et al. (2014), who successfully estimate heterogeneous agent models and provide further empirical support for this line of research.[1] In particular, empirical evidence reveals that speculators' rule-selection behavior depends on the rules' past profitability, on the markets' price trends, and their misalignments.

[1] See Lux and Zwinkels (2018) for a comprehensive survey.

The basic structure of our approach and our main findings may be summarized as follows. Motivated by the aforementioned empirical observations, we develop a model in which speculators rely on trend following, contrarian and fundamental trading rules to determine their orders. Moreover, speculators switch between these rules with respect to past and future performance indicators. In particular, speculators opt for rules that have produced higher profits in the recent past. However, speculators also evaluate current market circumstances to find out which of their rules may be most suitable for the current trading period: speculators favor the fundamental trading rule if prices are more distant to fundamental values, while they prefer trend-following and contrarian trading rules if they observe significant price changes. In the first case, speculators are more convinced that prices will revert to the fundamental value; in the second case, they have greater trust in their technical trading signals. Finally, there is a market maker who adjusts prices according to speculators' order flow.

Within our model, speculators' rule selection behavior has a probabilistic nature. This kind of noise, which represents the only source of randomness in our model, has an intrinsic motivation and vanishes as the number of speculators increases. For an infinite number of speculators, the dynamics of our model is driven by a four-dimensional deterministic nonlinear map. Despite our general model setup, we analytically prove that it has a unique steady state in which prices mirror their fundamental value. In addition, the model's steady state becomes unstable due to a Neimark-Sacker bifurcation if speculators rely strongly on the trend-following trading rule. The dynamics is then characterized by a limit cycle, i.e. price trends tend to continue. However, the steady state may also become unstable due to a Flip bifurcation, giving rise to a period-two cycle and a permanent reversal of the current price trend. This is the case if speculators heavily rely on the contrarian or on the fundamental trading rule. Our analytical insights allow an explanation of why price changes may become unpredictable. When speculators start to switch more erratically between the trend-following and the contrarian trading rule, we begin to observe an increasingly intricate mixing of a continuation and reversal of the current price trend – up to the point where prices display a random walk-like path.

Overall, we find that our model is able to replicate the statistical properties of financial markets quite well. For a finite number of speculators, our model produces bubbles and crashes, excess volatility, fat-tailed return distributions, serially uncorrelated price changes and volatility clustering. In this sense, our model can be considered validated. As we will see, our model has allowed us to identify crucial driving forces behind these phenomena.

The remainder of our paper is organized as follows. In Section 2, we develop our model. In Section 3, we show that its dynamics is – for an infinite number of speculators – driven by a four-dimensional nonlinear deterministic map; we then present our main analytical results and illustrate the model's deterministic out-of-equilibrium behavior. In Section 4, we apply our model to the data and find that it is – for a finite number of speculators – able to reproduce the stylized facts of financial markets. In Section 5, we conclude our paper and highlight various avenues for future research.

2. The setup of our model

We consider a financial market that is populated by a market maker and N boundedly rational and heterogeneous speculators. The market maker adjusts prices with respect to speculators' excess demand. Speculators switch between trend-following, contrarian and fundamental trading rules to determine their orders. Since these rules predict a continuation of the price trend, a reversal of the price trend or a fundamental price correction, they are denoted by C, R and F. Speculators' rule-selection behavior has a probabilistic nature, and depends on past and future performance indicators. In particular, the fitness of a trading rule increases with its past profitability. Moreover, the fitness of the two technical trading rules increases with the strength of the current price signal, while the fitness of the fundamental trading rule increases with the size of the market's misalignments. Speculators' probabilistic rule-selection behavior represents the only type of randomness in our model. For an infinite number of speculators, the dynamics of our model is driven by a four-dimensional nonlinear deterministic map.

Let us start with a description of the market maker's behavior. Following Beja and Goldman (1980), Day and Huang (1990) and Farmer and Joshi (2002), the market maker adjusts log price P_t with respect to excess demand E_t, using the log-linear price adjustment rule

$$P_{t+1} = P_t + aE_t, \tag{1}$$

where a is a positive price adjustment parameter. Let W_t^C, W_t^R and W_t^F denote the market shares of speculators who rely on the trend-following, the contrarian and the fundamental trading rule, and let D_t^C, D_t^R and D_t^F represent the orders generated by these rules. Then speculators' population-weighted aggregate excess demand can be defined as

$$E_t = W_t^C D_t^C + W_t^R D_t^R + W_t^F D_t^F. \tag{2}$$

Accordingly, the market maker increases the price if the excess demand is positive, and vice versa. As is well known, such behavior implies that the market maker is risk-neutral and able to mediate all speculative orders.[2]

[2]Franke (2009) considers a model in which the market maker and speculators are risk-averse, and they

Technical analysis (Lo et al. 2000) seeks to extract trading signals out of past price movements. Within our model, speculators can choose between a trend-following and a contrarian trading rule. The trend-following trading rule is based on the hypothesis that the current price trend will continue in the same direction, while the contrarian trading rule assumes that the price trend will reverse its direction. We formalize the two technical trading rules in a rather general way by stating that

$$D_t^C = \Phi^C(v_t) \tag{3}$$

and

$$D_t^R = \Phi^R(-v_t), \tag{4}$$

where $v_t := P_t - P_{t-1}$ represents the actual price trend. Note that functions Φ^C and Φ^R are such that $\Phi^C(0) = \Phi^R(0) = 0$, $\Phi_v^C > 0$ and $\Phi_{-v}^R > 0$, i.e. the two technical trading strategies do not generate any trading signals in the absence of price trends, while the trend-following (contrarian) trading rule produces buying (selling) signals if price trends are positive.

Fundamental analysis (Graham and Dodd 1951) postulates that prices revert to their fundamental value, and their signals suggest buying (selling) in undervalued (overvalued) markets. We capture this trading philosophy by writing

$$D_t^F = \Phi^F(m_t), \tag{5}$$

where $m_t := F - P_t$ measures the price deviation from the fundamental value. To prevent exogenous noise from entering our model, we set the log fundamental value F at a constant rate. In line with the basic principles of fundamental analysis, we assume for function Φ^F that $\Phi^F(0) = 0$ and $\Phi_m^F > 0$. Hence, fundamentalists do not trade if the price mirrors its fundamental value, but become increasingly aggressive as the market's mispricing grows.

Speculators' rule-selection behavior depends on past and future performance indicators. Brock and Hommes (1998) stress that speculators switch between trading rules with respect to the rules' past profitability. In doing so, speculators hope that a trading rule's past profitability is a good indicator of its future profitability. However, speculators also monitor current market conditions to identify profitable trading rules. Lux (1995) argues that speculators would be more inclined to use technical analysis if stronger price trends were present, while Franke and Westerhoff (2012) stress that speculators favor fundamental analysis in periods with major

actively manage their inventory. He reports that, as long as market participants do not control their inventory too aggressively, their positions remain bounded and the effects on the dynamics are negligible. For simplicity, we therefore refrain from such model complications, as is the case with almost all models in this area.

misalignments. Clearly, technical analysis is based on trends, and speculators may put greater trust in technical analysis recommendations if there are significant price signals. And since every bubble eventually bursts, speculators become increasingly convinced that a fundamental price correction is about to set in as mispricing grows. To take these considerations into account, we express the fitness of the trading rules by

$$A_t^i = \Theta^i(g_t^i, m_t, v_t), \qquad i = C, R, F, \tag{6}$$

where $g_t^i := (Exp[P_t] - Exp[P_{t-1}])D_{t-2}^i$ and $\Theta^i(0,0,0) = 0$. Note that g_t^i reflects the past profitability of the three trading rules. As in Westerhoff and Dieci (2006), we assume that orders submitted in period $t-2$ are executed at the price in period $t-1$. The last observable profit then accomplished by a trading rule depends on the price realized in period t. Of course, functions Θ^i are such that $\Theta^i_{g^i} > 0$. Since speculators exhibit an increasing preference for fundamental analysis as the price deviates from its fundamental value while they prefer technical analysis as the current price change increases, we further assume that $\Theta^C_{|m|} < 0$, $\Theta^R_{|m|} < 0$, $\Theta^F_{|m|} > 0$, $\Theta^C_{|v|} > 0$, $\Theta^R_{|v|} > 0$ and $\Theta^F_{|v|} < 0$.

Inspired by Brock and Hommes (1997, 1998), we formalize speculators' discrete choice probabilities by

$$prob_t^i = \frac{Exp[hA_t^i]}{Exp[hA_t^C] + Exp[hA_t^R] + Exp[hA_t^F]}, \qquad i = C, R, F. \tag{7}$$

The probability that a speculator will follow the trend-following, the contrarian or the fundamental trading rule depends positively on the trading rules' current fitness. Parameter $h \geq 0$ is called the intensity of choice, and measures how sensitively a speculator reacts to differences in the fitness of trading rules. Note that the choice sensitivity of a speculator increases with parameter h, as can easily be illustrated by two extreme examples. For $h = 0$, a speculator randomly picks a trading rule, i.e. $prob_t^C = prob_t^R = prob_t^F = 1/3$, while for $h \to \infty$, a speculator always picks the trading rule with the highest fitness, even if the fitness advantage is only small.

Since there are N speculators in total, it follows that the number of trend followers, contrarians and fundamentalists is trinomially distributed with

$$(N_t^C, N_t^R, N_t^F) \sim \mathcal{T}(N, prob_t^C, prob_t^R, prob_t^F). \tag{8}$$

Note that the mean numbers of speculators who rely on the trend-following, the contrarian and the fundamental trading rules amount to $Nprob_t^C$, $Nprob_t^R$ and $Nprob_t^F$, while the corresponding variances are given by $Nprob_t^C(1 - prob_t^C)$, $Nprob_t^R(1 - prob_t^R)$ and $Nprob_t^F(1 - prob_t^F)$.

Obviously, the market shares of the three groups of speculators are defined as

$$W_t^i = \frac{N_t^i}{N}, \qquad i = C, R, F, \tag{9}$$

which completes the description of our model.

3. Deterministic dynamics

Assuming that there is an infinite number of speculators allows us to obtain a number of insights which help us to understand the model's functioning. In Section 3.1, we discuss our main analytical results. In Section 3.2, we analyze the model's deterministic out-of-equilibrium behavior.

3.1. Analytical results

Note that the model's intrinsic noise vanishes as the number of speculators approaches infinity. In fact, if $N \to \infty$, we obtain $W_t^i = \frac{Exp[hA_t^i]}{Exp[hA_t^C]+Exp[hA_t^R]+Exp[hA_t^F]}$ for $i = C, R, F$. The model's price evolution is then due to a fourth-order nonlinear difference equation, say $P_{t+1} = \Omega(P_t, P_{t-1}, P_{t-2}, P_{t-3})$, and the introduction of the auxiliary variables $X_{t+1} = P_t$, $Y_{t+1} = X_t$ and $Z_{t+1} = Y_t$ enables us to express the model's dynamical system by the four-dimensional nonlinear map

$$S : \begin{cases} P_{t+1} = P_t + a(W_t^C \cdot \Phi^C(P_t - X_t) + W_t^R \cdot \Phi^R(X_t - P_t) + W_t^F \cdot \Phi^F(F - P_t)) \\ X_{t+1} = P_t \\ Y_{t+1} = X_t \\ Z_{t+1} = Y_t \end{cases}, \tag{10}$$

where

$$W_t^i = \frac{Exp[hA_t^i]}{Exp[hA_t^C] + Exp[hA_t^R] + Exp[hA_t^F]}, \qquad i = C, R, F,$$
$$A_t^C = \Theta^C((Exp[P_t] - Exp[X_t]) \cdot \Phi^C(Y_t - Z_t), F - P_t, P_t - X_t),$$
$$A_t^R = \Theta^R((Exp[P_t] - Exp[X_t]) \cdot \Phi^R(Z_t - Y_t), F - P_t, P_t - X_t)$$

and

$$A_t^F = \Theta^F((Exp[P_t] - Exp[X_t]) \cdot \Phi^F(F - Y_t), F - P_t, P_t - X_t).$$

Straightforward computations reveal that the model admits a unique steady state in which prices reflect their fundamental value. Thus, we denote the unique steady state as the fundamental steady state $FSS = (P^*, X^*, Y^*, Z^*) = (F, F, F, F)$. Let us define the partial derivatives of functions Φ^C, Φ^R and Φ^F at the FSS by $\Phi_v^C(0) := b > 0$, $\Phi_{-v}^R(0) := c > 0$ and

$\Phi^F_m(0) := d > 0$. As we will see, the trading aggressiveness of trend-followers, contrarians and fundamentalists turns out to be crucial for the local asymptotic stability of the FSS. Despite our general model setup, we are able to state the following proposition.

Proposition 1. *The* $FSS = (P^*, X^*, Y^*, Z^*) = (F, F, F, F)$ *is locally asymptotically stable if and only if*

$$-\frac{3}{a} + \frac{d}{2} < b - c < \frac{3}{a}, \tag{11}$$

where violation of the first inequality is associated with a Flip bifurcation, while violation of the second inequality results in a Neimark-Sacker bifurcation.

Proof. The local asymptotic stability of the FSS depends on the eigenvalues of the Jacobian. To build the Jacobian, we need to compute the derivatives of P_{t+1}, X_{t+1}, Y_{t+1} and Z_{t+1} with respect to each variable. Applying the chain rule yields

$$\frac{\partial P_{t+1}}{\partial P_t} = 1 + a\Big(\frac{\partial W^C_t}{\partial P_t} D^C_t + W^C_t \frac{\partial D^C_t}{\partial P_t} + \frac{\partial W^R_t}{\partial P_t} D^R_t + W^R_t \frac{\partial D^R_t}{\partial P_t} + \frac{\partial W^F_t}{\partial P_t} D^F_t + W^F_t \frac{\partial D^F_t}{\partial P_t}\Big),$$

where $D^C_t = \Phi^C(P_t - X_t)$, $D^R_t = \Phi^R(X_t - P_t)$ and $D^F_t = \Phi^F(F - P_t)$. At the FSS, we have $D^i_t = \Phi^i(0) = 0$, $A^i_t = \Theta^i(0,0,0) = 0$ and $W^i_t = \frac{1}{3}$ for $i = C, R, F$. Moreover, recall that the partial derivatives of Φ^C, Φ^R and Φ^F at the FSS are given by $\Phi^C_v(0) := b$, $\Phi^R_{-v}(0) := c$ and $\Phi^F_m(0) := d$. Thus, we obtain

$$\frac{\partial P_{t+1}}{\partial P_t}\Big|_{FSS} = 1 + a\Big(\frac{1}{3}b - \frac{1}{3}c - \frac{1}{3}d\Big).$$

Symmetrically, we get

$$\frac{\partial P_{t+1}}{\partial X_t}\Big|_{FSS} = a\Big(-\frac{1}{3}b + \frac{1}{3}c\Big), \quad \frac{\partial P_{t+1}}{\partial Y_t}\Big|_{FSS} = 0, \quad \frac{\partial P_{t+1}}{\partial Z_t}\Big|_{FSS} = 0.$$

Hence, the Jacobian at the fundamental steady state can be written as

$$J(FSS) = \begin{pmatrix} 1 + a\Big(\frac{b}{3} - \frac{c}{3} - \frac{d}{3}\Big) & a\Big(-\frac{b}{3} + \frac{c}{3}\Big) & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

from which we obtain the characteristic polynomial

$$\mathcal{P}(\lambda) = \lambda^2\Big(\lambda^2 - \lambda\big(1 + \frac{a}{3}(b - c - d)\big) + \frac{a}{3}(b - c)\Big) = 0.$$

As can be seen, we have $\lambda_{1/2} = 0$, which implies that the local asymptotic stability of the fundamental steady state depends on the characteristic roots of the remaining second-degree polynomial. These are jointly smaller than 1 in modulus if (i) $\mathcal{P}(1) > 0$, (ii) $\mathcal{P}(-1) > 0$ and (iii) $1 - \mathcal{P}(0) > 0$, which correspond to the (i) Fold, (ii) Flip and (iii) Neimark-Sacker bifurcation, respectively (see, e.g. Medio and Lines 2001). Since condition (i) is always fulfilled, the FSS is locally asymptotically stable if inequalities (ii) and (iii) simultaneously hold, i.e. if and only if

$$-\frac{3}{a} + \frac{d}{2} < b - c < \frac{3}{a}.$$

□

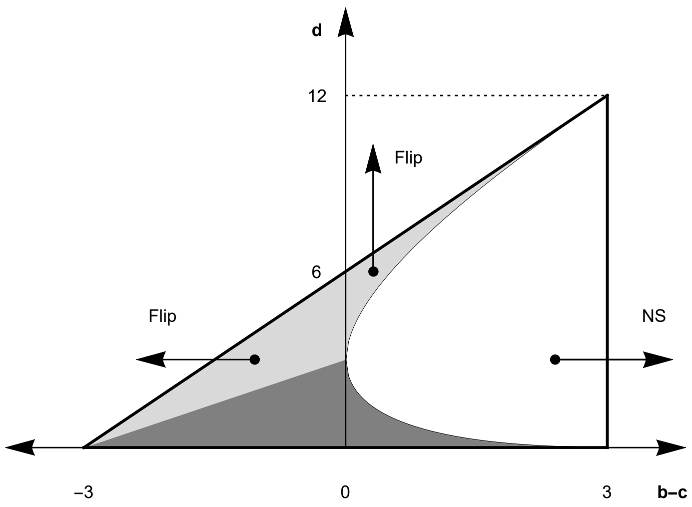

Figure 1: Local stability of the FSS. The triangle shows combinations of d and $b-c$ for which the FSS is locally asymptotically stable (white: cyclical adjustment, light gray: alternating adjustment, dark gray: monotonic adjustment), assuming that $a = 1$. The black lines represent the stability borders $b - c = -3 + \frac{d}{2}$ and $b - c = 3$. The arrows indicate possible Flip and Neimark-Sacker bifurcation routes.

To illustrate our analytical results, we set $a = 1$ and plot the stability conditions in Figure 1. The inner part of the triangle represents all d and $b - c$ parameter combinations for which the FSS is locally asymptotically stable. In addition, the white, light gray and dark gray areas imply a cyclical, an alternating and a monotonic price adjustment, respectively.[3] Our analysis reveals that the FSS becomes unstable due to a Neimark-Sacker bifurcation if speculators react strongly to the trading signals generated by the trend-following rule, that is, if $b > 3 + c$. In such a situation, we observe the birth of a limit cycle, implying that the current price

[3]Note that the eigenvalues of the Jacobian at the FSS are complex if and only if $Tr^2 - 4Det < 0$, i.e. $3 + b - c - 2\sqrt{3}\sqrt{b-c} < d < 3 + b - c + 2\sqrt{3}\sqrt{b-c}$. If this condition does not hold, monotonic (alternating) dynamics emerges if $Tr \leq 0$ ($Tr > 0$), i.e. if $d \leq 3 + b - c$ ($d > 3 + b - c$).

trend tends to continue. In contrast, the FSS becomes unstable by a Flip bifurcation if speculators react strongly to the trading signals generated by the contrarian trading rule, that is, if $c > 3 + b - \frac{d}{2}$. Due to the emergence of a period-two cycle, the dynamics is then characterized by a permanent reversal of the current price trend. These analytical insights are quite valuable for our understanding of why price changes may become serially uncorrelated. Suppose that speculators begin to switch rather unsystematically between the trend-following and the contrarian trading rule. Periods in which the trend-following trading rule dominates the market, causing a continuation of the current price trend, then start to erratically alternate with periods in which the contrarian trading rule dominates the market, causing a reversal of the current price trend. Together, these forces generate – as speculators switch more and more erratically between these rules – a random walk-like price evolution. For completeness, we mention that the FSS may also become unstable if speculators react strongly to the trading signals triggered by the fundamental trading rule. In particular, we observe a Flip bifurcation and the onset of a period-two cycle if $d > 6 + 2(b - c)$.

3.2. Out-of-equilibrium behavior

To broaden our perspective, we now explore the model's deterministic out-of-equilibrium behavior. For this purpose, we need to specify the orders generated by the trend-following, contrarian and fundamental trading rules, and formalize them by

$$D_t^C = \mu^C ArcTan[\frac{b}{\mu^C}(P_t - P_{t-1})], \tag{12}$$

$$D_t^R = \mu^R ArcTan[\frac{c}{\mu^R}(P_{t-1} - P_t)] \tag{13}$$

and

$$D_t^F = \mu^F ArcTan[\frac{d}{\mu^F}(F - P_t)], \tag{14}$$

where $\mu^C := \frac{2}{\pi}\beta$, $\mu^R := \frac{2}{\pi}\gamma$, $\mu^F := \frac{2}{\pi}\delta$ and $\beta, \gamma, \delta > 0$. Note that (12), (13) and (14) are sigmoid functions with a symmetric S-shaped form, bounded between $[-\beta, \beta]$, $[-\gamma, \gamma]$ and $[-\delta, \delta]$, respectively.[4] Also consistent with our general model framework, we specify the attractiveness of the three trading rules as

$$A_t^C = e(Exp[P_t] - Exp[P_{t-1}])D_{t-2}^C - f(F - P_t)^2 + g(P_t - P_{t-1})^2, \tag{15}$$

[4]Similar trading rules, albeit with larger lags, are used, for instance, in Chiarella et al. (2006).

$$A_t^R = e(Exp[P_t] - Exp[P_{t-1}])D_{t-2}^R - f(F - P_t)^2 + g(P_t - P_{t-1})^2 \tag{16}$$

and

$$A_t^F = e(Exp[P_t] - Exp[P_{t-1}])D_{t-2}^F + f(F - P_t)^2 - g(P_t - P_{t-1})^2, \tag{17}$$

where parameters $e, f, g > 0$ control how the three fitness components affect the overall attractiveness of the trading rules.

Let us start our numerical investigation by discussing the model's main bifurcation routes. In the left panels of Figure 2, we show bifurcation diagrams in which parameters b, c and d are varied as indicated on the axes, while the other parameters are set to $a = b = \beta = c = \gamma = d = \delta = e = f = g = h = 1$ and $F = 0$. As predicted by our analytical results, we observe a Neimark-Sacker bifurcation at $b = 4$ in the top left panel. As the amplitude of the cycles grows with increasing values of b, we can conclude that a financial market in which speculators rely more strongly on the trend-following trading rule becomes destabilized. The center left panel of Figure 2 reveals a Flip bifurcation and the birth of a period-two cycle at $c = 3.5$. Therefore, a financial market also becomes destabilized if speculators rely more heavily on the contrarian trading rule. The bottom left panel of Figure 2 reveals another route toward instability: endogenous dynamics may set in if speculators bet strongly on a fundamental price correction. In fact, a Flip bifurcation occurs at $d = 6$. As can be seen, the period-two cycle describes increasingly dramatic price fluctuations as parameter d further increases. The right panels of Figure 2 repeat these numerical experiments for $\beta = 1.2$ (top), $\gamma = 6$ (center) and $\delta = 10$ (bottom). Recall that parameters β, γ and δ limit the maximum order size of the trend-following, the contrarian and the fundamental trading rules. In this sense, higher values of β, γ and δ indicate more aggressive speculators, and increase the amplitude of the dynamics.

The six panels of Figure 3 depict the final dynamics of the six panels of Figure 2 in the time domain. Hence, we show in the top, center and bottom row of Figure 3 the model dynamics of the top, center and bottom row of Figure 2 for $b = 9$, $c = 9$ and $d = 9$, respectively. As can be seen from the top row, the Neimark-Sacker bifurcation generates a quasi-period motion. Economically, this implies that speculators who rely heavily on the trend-following trading rule tend to enforce a continuation of the current price trend. Moreover, the amplitude of the cycles increases if trend-followers trade more aggressively, i.e. if β increases. However, this differs from the situation depicted in the second row. When speculators pay greater attention to the contrarian trading rule, we observe a period-two cycle for $\gamma = 1$ (left panel), while the dynamics shows a more intricate price reversal pattern for $\gamma = 6$ (right panel). Interestingly, these price reversals may occur above or below the fundamental value, i.e. the fundamental

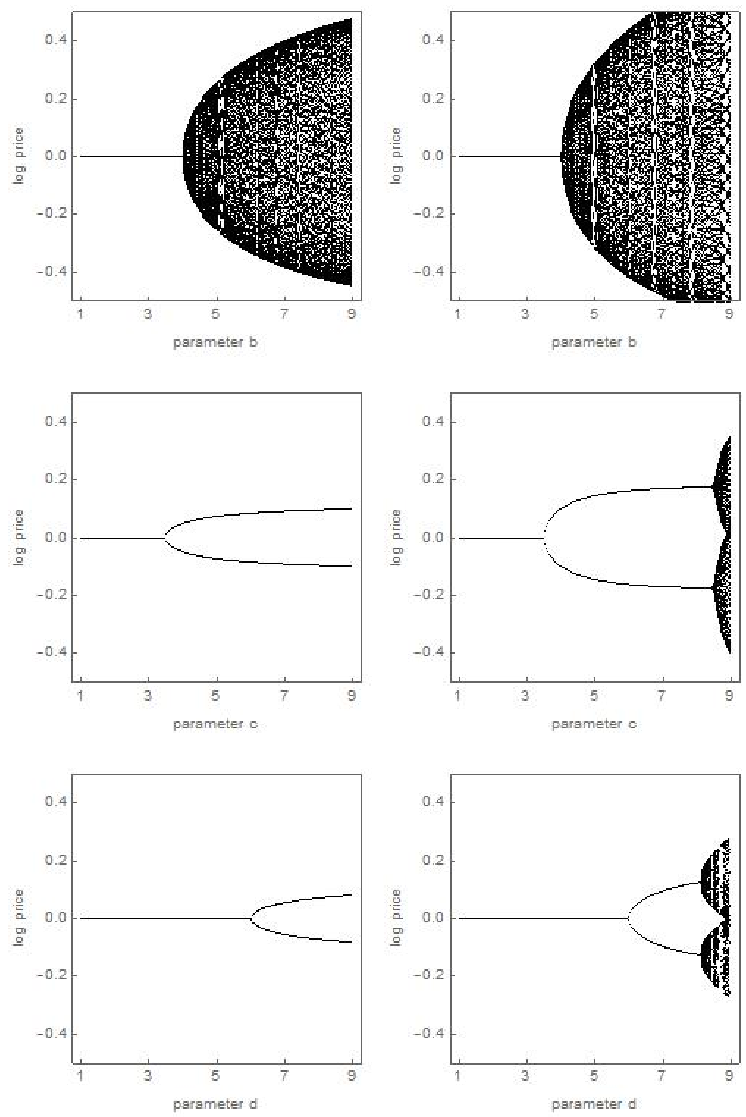

Figure 2: Bifurcation diagrams for parameters b, c and d. While the bifurcation parameters are increased as indicated on the axes, we assume for the other parameters that $a = b = \beta = c = \gamma = d = \delta = e = f = g = h = 1$ and $F = 0$. The right panels show the same, except that $\beta = 1.2$ (top), $\gamma = 6$ (center) and $\delta = 10$ (bottom).

value is not crossed every period.

The left (right) panel of the bottom row of Figure 3 is based on $d = 9$ and $\delta = 1$ ($\delta = 10$). It becomes obvious from the right panel that our model can also produce complex price dynamics.[5]

[5]Our model may also produce other and much more intricate dynamic phenomena, including chaotic motion and coexisting attractors. For instance, a locally stable fixed point may coexist with an attracting limit cycle, implying that tiny changes in the model parameters or exogenous noise may lead to abrupt changes in the dynamics. Moreover, two locally stable period-two cycles may coexist, one located above and the other below

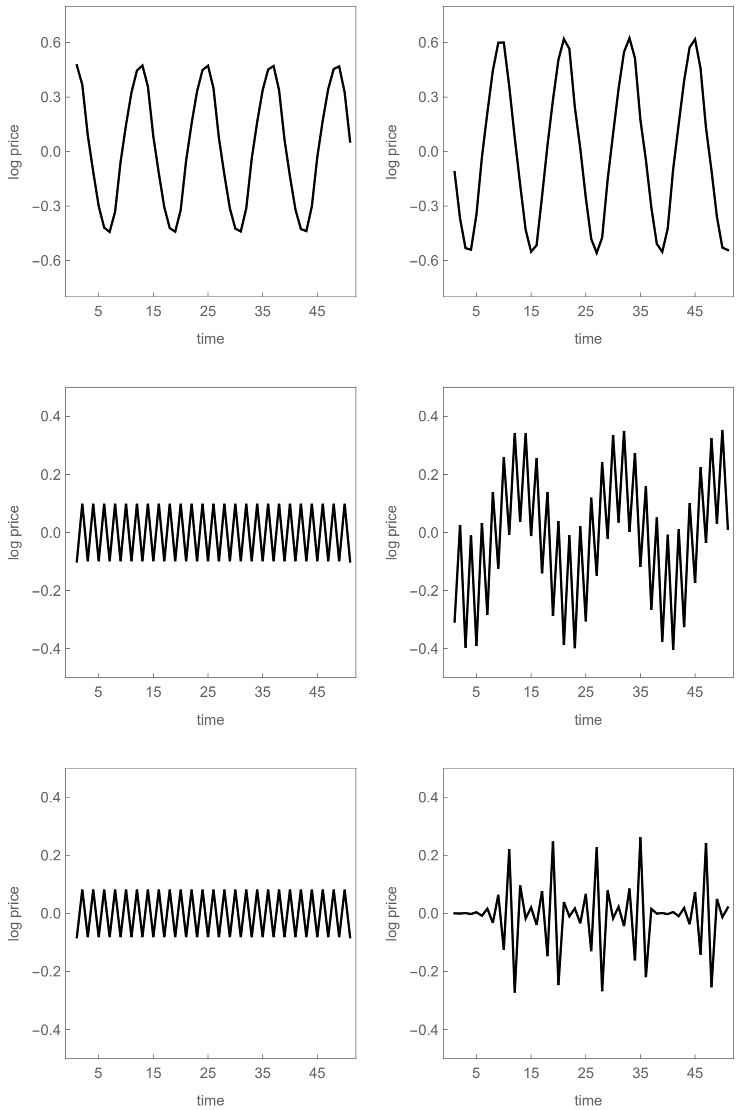

Figure 3: Examples of price continuation and price reversal dynamics. The panels visualize the final dynamics of the panels in Figure 2 in the time domain. Since the panels correspond to the ones in Figure 2, they are based on the same parameter settings, except that $b = 9$ (top), $c = 9$ (center) and $d = 9$ (bottom).

Note that the simulation run is based on $b = \beta = c = \gamma = 1$, i.e. the price impacts of the trend-following and the contrarian trading rules cancel each other out at the steady state. Out of equilibrium, however, the price impact of these rules is not zero. Depending on the relative size of the market shares of these rules, we may observe a continuation or a reversal of the current price trend. This is important: should speculators switch erratically between

the fundamental value. The model can then produce persistent bull and bear market dynamics.

the trend-following and the contrarian trading rule, we may observe a very intricate mixing of these two regimes, resulting in a random walk-like price evolution. The next section reveals that this is the case when the number of speculators becomes finite.

4. Stochastic dynamics

Setting the number of speculators to a finite value – as we do in the remainder of our paper – implies that speculators' probabilistic rule-selection behavior adds intrinsic noise to the dynamics. In Section 4.1, we first show that the resulting stochastic dynamics matches a number of important stylized facts of financial markets. In Section 4.2, we then explain in more detail how this comes about.

4.1. Matching the stylized facts of stock markets

Let us start with a brief recap of the main stylized facts of financial markets. As made clear by Shiller (2015), Lux and Ausloos (2002) and Cont (2001), financial markets are characterized by (i) bubbles and crashes, (ii) excess volatility, (iii) fat-tailed return distributions, (iv) serially uncorrelated returns and (v) volatility clustering. In Figure 4, we illustrate these properties for the S&P500. The underlying dataset, downloaded from Refinitiv Datastream, runs from 1964 to 2018, and comprises 13802 daily observations. The top panel of Figure 4 presents the time evolution of the S&P500. Despite its gradual upwards trend, a number of distinct bubbles and crashes are visible. The panel below shows the corresponding return time series. Note that the volatility of the S&P500 is quite high. For instance, the average absolute return of the S&P500 is given as 0.69 percent, i.e. the S&P500 changes by almost 0.7 percent per day. Moreover, there are a number of larger returns, and volatility varies over time. In the central panel of Figure 4, we compare the log probability density function of normalized S&P500 returns (black dots) with that of standard normally distributed returns (gray line). Obviously, the former contains more probability mass in the center, less probability mass in the shoulders, and again more probability mass in the tails than the latter. The Hill tail index estimator allows us to quantify this property. Taking the largest 5 percent of the observations into account, we find a tail index of about 3.01, suggesting that the fourth moment of the distribution of returns does not exist. The penultimate panel of Figure 4 depicts the autocorrelation coefficients of raw returns for the first 100 lags, together with their 95 percent confidence bands. The autocorrelation coefficients of raw returns are relatively low, and we can consider them insignificant. As a result, the S&P500 is barely predictable, and closely resembles a random walk. The bottom panel of Figure 4 depicts the autocorrelation function of absolute returns for the first 100 lags.

Without question, the autocorrelation coefficients of absolute returns are highly significant and indicate that volatility is persistent for more than 100 trading days.

For brevity, we only present a single representative simulation run with 13750 observations in our paper. Assuming 250 trading days per year corresponds to a time span of 55 years, which is comparable to the length of our empirical dataset. A comparison of Figure 4 and Figure 5 reveals that our model is able to match a number of important stylized facts of financial markets. To be precise, the first panel of Figure 5 displays the development of log prices. As can be seen, the log price oscillates around its log fundamental value, i.e. lasting periods of overvaluation alternate with lasting periods of undervaluation.[6] The second panel of Figure 5 presents the corresponding return time series, and witnesses a high volatility. The average absolute return of this time series amounts to 0.61, comparable to the value we received for the S&P500. The third panel of Figure 5 documents the fact that the distribution of normalized returns deviates from the normal distribution, as is the case for almost all financial markets. The tail index for this time series is estimated at 3.2, a value near most empirical observations. According to the fourth panel of Figure 5, returns are serially uncorrelated, implying that it is difficult to predict the model's future price path. Finally, the fifth panel of Figure 5 indicates that volatility tends to cluster. While it is already apparent from the second panel of Figure 5 that calm periods alternate with turbulent periods, we note that the autocorrelation function of simulated absolute returns is close to the one we obtained for the S&P500.

A few comments with respect to the calibration and performance of our model are in order. So far, we have not exploited the full flexibility of our model. Instead, we initially fixed $a = 1$, $h = 1$, $F = 0$ and $N = 2500$; we also introduced the restrictions $b = c$ and $\beta = \gamma = \delta$, leaving us six degrees of freedom. After a trial-and-error calibration exercise, we then decided to set $b = c = 2000$, $d = 0.002$, $\beta = \gamma = \delta = 1.1$, $e = 0.05$, $f = 50$ and $g = 2000$. Further experiments (not depicted) revealed that the model performance does not react overly sensitively with respect to changes in the above parameter setting. We note that we also conducted an in-depth Monte Carlo study, which permitted us to conclude that our calibrated model systematically replicates the main statistical properties of financial markets.

[6] Identifying bubbles and crashes in real financial markets requires a good understanding of the markets' fundamental values. Following Shiller's (2015) famous notion of the S&P500's fundamental value, Schmitt and Westerhoff (2017) show that the distortion of the S&P500, i.e. the log distance between the S&P500 and its fundamental value, displays a bimodal distribution. Without going into too much detail, the same is true for the simulated dynamics of our stochastic model. Periods in which the market is either overvalued or undervalued are more likely than periods in which the market remains near its fundamental value.

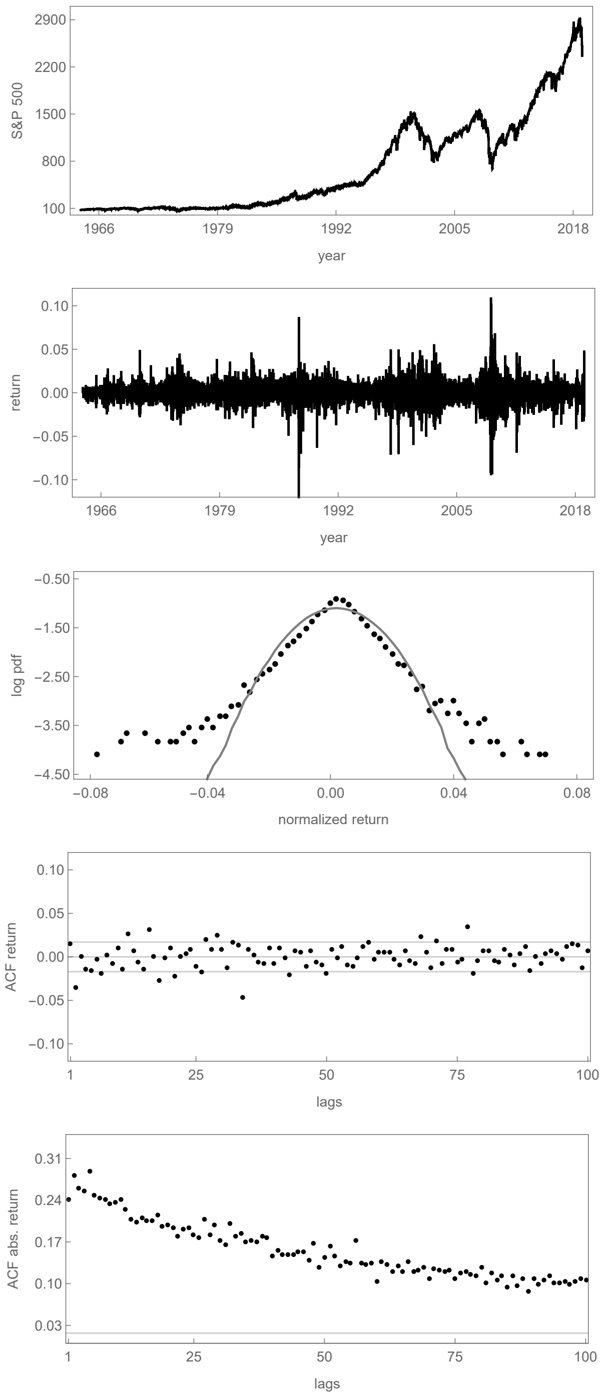

Figure 4: Stylized facts of financial markets. The panels show the time evolution of the S&P500, the corresponding returns, the log probability density function of normalized returns, the autocorrelation function of raw returns and the autocorrelation function of absolute returns, respectively. The data series runs from 1964 to 2018, and comprises 13802 daily observations.

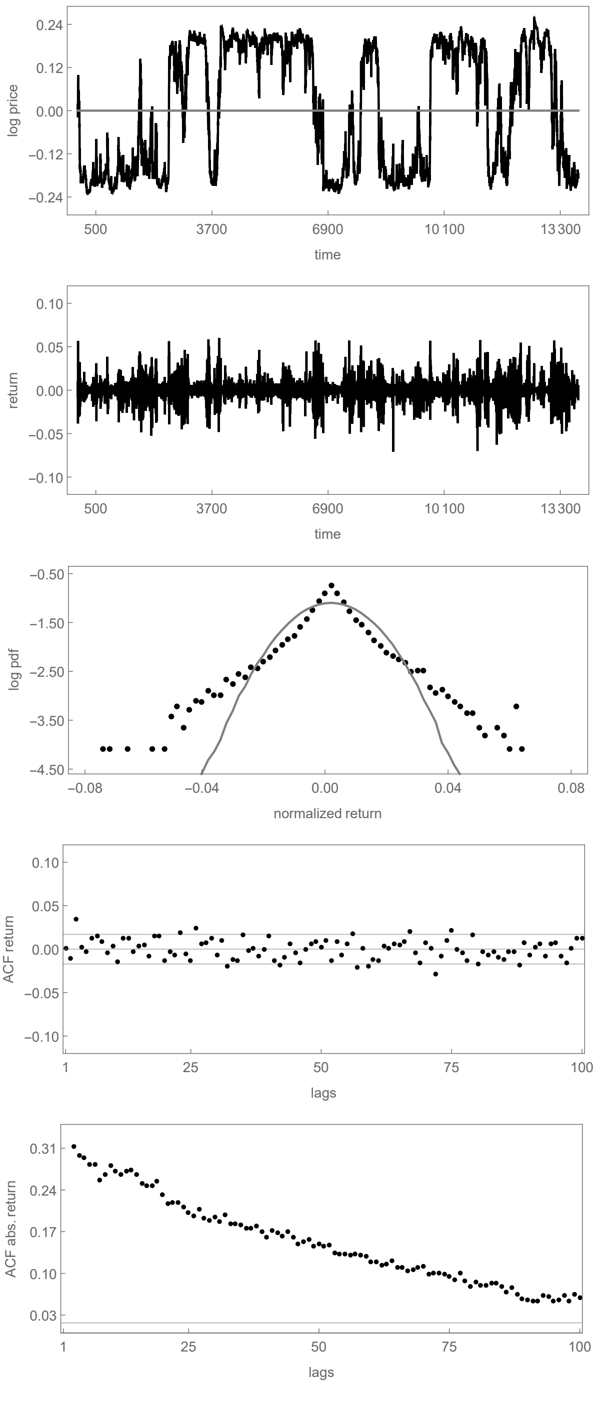

Figure 5: Matching the stylized facts of financial markets. The panels show the time evolution of log prices, the corresponding returns, the log probability density function of normalized returns, the autocorrelation function of raw returns and the autocorrelation function of absolute returns, respectively. The simulated time series comprises 13750 observations. The parameter setting is reported in Section 4.1.

However, future work may try to fine tune or even to estimate the parameters of our model. For our explanatory purposes, the model's performance already appears to us to be sufficient.

4.2. Functioning of the stochastic model

In Figure 6, we present a snapshot of the model dynamics to explain how the model functions. The panels show from top to bottom the time evolution of log prices, the corresponding returns, and the fractions of trend followers (black), fundamentalists (white) and contrarians (gray), respectively. The simulated time series comprises 500 observations, and thus represents a time span of about two years. As the figure shows, a dramatic boom-bust cycle, associated with a volatility outburst and dominance by technical traders, occurs between periods 150 and 250. Obviously, the trading behavior of speculators who rely on the trend-following and the contrarian trading rules may quickly drive prices away from fundamental values, while the trading behavior of speculators who use the fundamental trading rule establishes a relatively slow mean reversion. In fact, the market is undervalued most of the time during this period. However, speculators' trading behavior may not only cause severe bubbles and crashes. Since the fundamental value is constant, prices are excessively volatile.

In our view, the most striking property of our model is that it can produce serially uncorrelated price changes. According to our model calibration, the impacts of the trend-following and the contrarian trading rules are balanced on average. In a given period, however, trend followers may dominate the market. As predicted by our analytical results, their trading behavior then tends to extend the current price trend. In another period, contrarians may dominate the market and, again in line with our analytical results, cause a reversal of the current price trend. Due to an erratic mixing of these two opposing regimes, we observe a random walk-like price evolution. The reason why this is the case is as follows. Speculators' rule-selection behavior depends on past profits, price signals and misalignments. Since prices follow a random walk, none of the rules produces systematic profits. In some periods, the trend-following trading rule outperforms the contrarian trading rule; while in other periods, it is the other way around. Essentially, the profit-dependent fitness component therefore acts like a random variable on the rules' total fitness. Moreover, significant price signals make the trend-following and the contrarian trading rules equally more popular, while higher misalignments make both rules equally less popular.

Occasionally, however, there is a more substantial mismatch between the market impact of trend followers and contrarians. On such days, excess demand may escalate, prompting the market maker to adjust prices more strongly. Such an incident is visible shortly after period 250, i.e. when the bubble bursts. Note that volatility outbursts emerge if the market

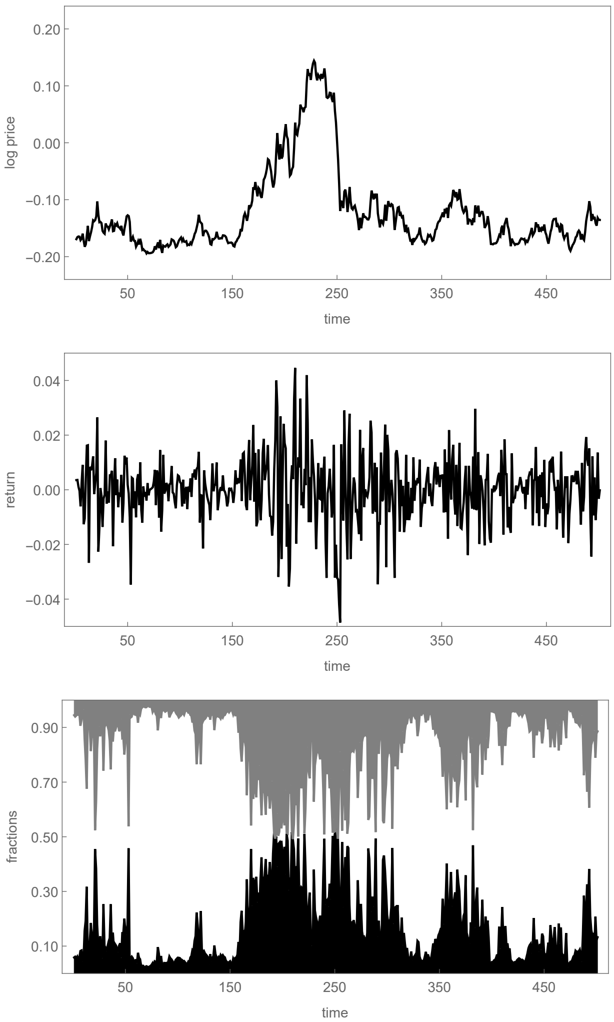

Figure 6: Functioning of the model. The panels show the time evolution of log prices, the corresponding returns, and the fractions of trend followers (black), fundamentalists (white) and contrarians (gray), respectively. The simulated time series comprises 500 observations, and is based on the same parameter setting as in Figure 5.

impact of fundamentalists decreases, either due to stronger price signals or lower misalignment. Speculators then favor trend-following and contrarian trading rules more strongly, which causes greater volatility, as can be seen between periods 150 and 350.

It seems worthwhile to mention that only a single exogenous shock is needed to set the model's dynamics in motion. Clearly, if the price matches its fundamental value for two consecutive trading periods, speculators no longer receive trading signals, and the price continues to stay equal to its fundamental value. However, once this relationship is distorted, everlasting dynamics, matching those of actual financial markets, are set in motion. This observation highlights once again the truly endogenous nature of the model's dynamics.

5. Conclusions

The goal of our paper is to develop a fairly general, analytically tractable and empirically motivated model to foster our understanding of how financial markets function. In particular, we seek to match a whole battery of stylized facts of financial markets with a sound economic model that does not rely on exogenous noise. The model we propose is populated by a market maker, who adjusts prices with respect to the excess demand, and boundedly rational speculators, who use trend-following, contrarian and fundamental trading rules to determine their orders. Speculators' probabilistic rule-selection behavior, reflecting the only type of (intrinsic) noise in our model, depends on past and future performance indicators. Speculators favor rules that have been more profitable in the past as well as an evaluation of current market circumstances. In this respect, speculators favor fundamental analysis when the market is heavily distorted, while they prefer technical analysis when the market exhibits clear price changes. Applying our model to the data reveals that it is able to produce bubbles and crashes, excess volatility, fat-tailed return distributions, uncorrelated price changes and volatility clustering, replicating five important stylized facts of financial markets. Against this backdrop, it is possible to state that our model may be regarded as validated.

A particularly hard challenge for models with heterogeneous interacting agents is to produce a random walk-like price evolution. Since our model rests on similar building blocks as many other models in the field, such as the market maker's price adjustment behavior, the trading activities of trend followers and fundamentalists, and/or the speculators' rule-selection behavior, the following question arises: which factors are responsible for our model's ability to match the stylized facts of financial markets, in particular its ability to generate serially uncorrelated price changes without relying on exogenous noise? The answer to this question is two-fold. First of all, our model takes greater account of the trading behavior of contrarians than other models. Since contrarians and trend followers trade in opposite directions, contrarians add a considerable amount of heterogeneity to our model. Although it is not the core focus of our paper, it should be noted that if one seeks to explain the high trading volume observed in many real financial markets, one has to develop a model in which agents are truly heterogeneous.

In our setup, the trading behavior of contrarians offsets at least some of the orders placed by trend followers and, at times, may even outweigh them. This is exactly why our model possesses two opposing dynamic regimes. In one regime, trend followers dominate the market and tend to reinforce the current price trend. In the other regime, contrarians control the market, and reverse the current price trend. Even for an infinite number of speculators, i.e. in a completely deterministic environment, our model is able to produce irregular dynamics. However, the full power of our model is unleashed when the number of speculators is (more realistically) set to a finite value. It is then that speculators' probabilistic rule-selection behavior kicks in and leads to even more erratic switching between the two regimes, establishing serially uncorrelated price changes. This model feature, absent in most related models, forms the second part of the above answer. As we have seen, speculators' intricate trading behavior also creates excess volatility, and may cause significant bubbles and crashes. Extreme price changes and volatility outbursts occur when speculators rely heavily on technical analysis.

We conclude our paper by highlighting three avenues for future research. First, it may be worthwhile to keep track of an individual speculator's behavior, e.g. to monitor their profits and trading positions. Some speculators may become wealthier over time and make a stronger impact on the market, while other speculators perish. Speculators may also be restricted in accumulating very large investment positions or in their ability to build a short position. With respect to the optimal size of a model, it seems to us that recent models by LeBaron (2011, 2012) are neither too simple nor too complicated. Second, one may also consider speculators' tendency to herd. As pointed out by Shiller (2015), herding may inflict a coordinated trading behavior and a substantial decrease in heterogeneity among speculators. Extreme price changes are often the result of unbalanced excess demand, i.e. a mismatch between buying and selling orders. Allowing for herding could increase the model's ability to produce really large price changes. Third, one may use our model to contemplate better regulation of financial markets. Our model reveals that there are constellations where either more trend followers or more contrarians would have a positive impact on market stability. Moreover, attempting to counter price changes in an effort to offset the trading signals of trend followers and contrarians, e.g. via interventions by a central authority, is difficult, since price changes are serially uncorrelated, and while a reduction of the market's misalignment may appear beneficial at first sight, it tends to increase the destabilizing market impact of technical analysis. Although the regulation of financial markets is a tricky issue (see Westerhoff and Franke 2018 for a survey), the instability of financial markets and the associated problems for the real economy show that it is important to address such issues. We hope that our paper will provide new insights and fresh impetus to continue this line of research.

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