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Horizontal product differentiation with limited attentive consumers*

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Abstract

We analyze the effects of consumers' limited attention on welfare in a model of horizontal product differentiation. We present a novel approach of modeling limited attention: an *attention radius*. Each consumer only notices goods that are within her attention radius, i.e., goods that are sufficiently similar to her preferred version of the good. Limited attention induces firms to differentiate their products in a way that is beneficial to consumers. In addition, prices may be lower under limited than under full attention. Consumer surplus and welfare are not maximized under full attention but increase for some degree of limited attention.

Keywords: Attention, Horizontal Product Differentiation, Hotelling, Price Discrimination.

JEL Codes: D43, D91, L13.

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1 Introduction

This article proposes a model of horizontal product differentiation that captures preference-dependent attention allocation of consumers. We investigate the effects of consumers' limited attention on consumer surplus, firms' profits, and overall welfare.

We construct a novel method of modeling limited attention. We model attention as a spotlight that only highlights the section of the product space around the consumer's preferred version of the good. For example, a consumer who prefers minivans only notices cars that are similar to minivans, like SUVs, and does not notice smaller cars, like compacts or roadsters, when she looks for a new car. Alternatively, a consumer who prefers blue focuses on blue t-shirts. Cyan or turquoise t-shirts also capture her attention as these colors are similar to blue, but red and brown t-shirts do not capture her attention as these colors are too far from blue. That means, we model attention allocation as preference-dependent; the consumer's preference primes her perception. Consumers only notice options that are similar to their preferred option and do not necessarily notice all available options.

Experiments on inattentional blindness demonstrate that it is reasonable to assume that consumers are not necessarily aware of all available goods in the market and that consumers are more likely to notice goods that are sufficiently similar to their target good. Inattentional blindness experiments show that by focusing on some events, people fail to perceive other events (see, e.g., Simons and Chabris, 1999; Most, Simons, Scholl, Jimenez, Clifford, and Chabris, 2001). In particular, inattentional blindness experiments show that similarity matters: For instance, if people focus on events in a particular color, they are more likely to notice other events if those events have the same color (e.g., Simons and Chabris, 1999; Most, Simons, Scholl, Jimenez, Clifford, and Chabris, 2001; Drew and Stothart, 2016).

We follow Hotelling (1929) in modeling horizontal product differentiation as a real line $[0, 1]$. Consumers are uniformly distributed on $[0, 1]$. The position $x \in [0, 1]$ of a consumer describes the consumer's preferred version of the good. Consumers are constrained in their attention: Each consumer only notices goods that are inside her *attention radius* κ. The *attention radius* highlights the section of the product space around the consumer's preferred version of the horizontally differentiated good, i.e., $[x-\kappa, x+\kappa]$. Figure 1 illustrates the attention radius of a consumer whose preferred version of the good is at $x \in [0,1]$. Suppose two versions of the good exist at y_1 and y_2. As y_1 is inside the consumer's attention radius, the consumer at x is aware of good 1. As y_2 is not inside the consumer's attention radius, the consumer at x is not aware of good 2.

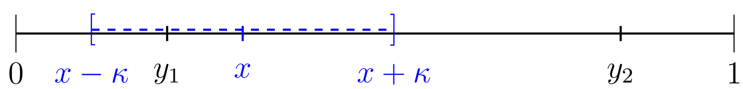

Figure 1: Example of the attention radius of the consumer at x.

In this article, we investigate the effects of such attention allocation of consumers on product differentiation. We follow d'Aspremont, Gabszewicz, and Thisse (1979) in modeling transportation costs as quadratic. In our model, transportation costs describe the disutility of consumers who consume a non-optimal good. We analyze the strategic considerations of two firms. Our analysis consists of two parts. In the first part, we assume that firms have to sell the good at an exogenously fixed price to derive the direct effects of consumers' limited attention on product differentiation. Then, firms independently and simultaneously choose the optimal location in the product space to maximize profits. We demonstrate that some degree of limited attention can have positive implications for consumers and overall welfare. In contrast to the standard Hotelling model where firms locate at the median consumer (see, e.g., Tirole, 1988), we show that firms choose to further differentiate their products. For lower levels of attention, the attention radii of consumers induce firms to locate closer to the efficient locations. Otherwise they miss demand. Thus consumers benefit from (some degree of) limited attention.

In the second part of the analysis, we allow firms to choose locations and prices. Firms, then, play a two-stage game: In the first stage, firms, independently and simultaneously, choose their locations in the product space. In the second stage, firms observe the location of their competitor and, independently and simultaneously, choose prices. Subsequently, consumers make a consumption decision. Three groups of consumers exist: Consumers who notice neither, one, or both firms. Firms compete for the consumers who notice both firms, but are monopolists for consumers who notice only one firm. We assume that a firm can price discriminate between consumers who notice only one firm and consumers who notice both firms. This captures situations where sellers can infer whether consumers are aware of competitors.

We show that very low levels of attention are not beneficial to consumers: Firms act as monopolists for all consumers, who then pay a high monopoly price. However, full attention is also not optimal for consumers. Under full attention, firms maximally differentiate their products and exploit this market power by setting higher prices. There exist intermediate levels of attention, where consumers pay lower prices than under full attention. Under these intermediate levels of attention, firms locate closer to the efficient locations than under full attention. This effect of limited attention on product differentiation also prevails under low levels of attention. Thus even under low levels of attention, consumers at least benefit on average from better product differentiation. Overall then, full attention is not optimal for consumers, instead consumer surplus is higher under some degree of limited attention.

The remainder of the article is structured as follows. Section 2 provides an overview of the related literature. Section 3 introduces the model. Section 4 discusses the results if prices are exogenously fixed and contrasts these results with the standard model with fully attentive consumers. In section 5, we allow firms to set prices and locations and analyze the resulting subgame-perfect equilibrium and the resulting welfare. Section 6 discusses the results and concludes. All proofs are in the appendix.

2 Related Literature

Horizontal product differentiation is an extensively discussed topic in economics and, although limited attention is a growing strand of the economic literature, few articles discuss limited attention in the context of a Hotelling (1929) model. Exceptions are Schultz (2004) and Polo (1991). Yet, these articles exogenously distinguish between attentive and inattentive consumers. The uninformed consumers are inattentive to, for example, prices and/or locations (e.g., Polo, 1991; Schultz, 2004).[1] Then, instead of making a consumption choice with perfect information, inattentive consumers form expectations (Schultz, 2004) or buy from the nearest or cheapest firm (Polo, 1991). These models show that the fraction of consumers who are inattentive, distinctly influences market outcomes. Schultz (2004), for instance, shows that product differentiation, prices, and profits decrease in the number of attentive consumers. Yet, in these models the distinction who is informed and who is uninformed is random and consumers are generally aware of the existence of all firms. However, the distinction who is attentive and who is inattentive can also arise endogenously because of horizontal product differentiation. We add to this strand of the literature by analyzing this preference-dependent allocation of attention.

In addition, in our model, consumers are only aware of firms inside their attention radius. With this modeling choice, we also add to the literature on consideration sets by proposing a novel formation criteria for consideration sets in models of horizontal product differentiation. Generally, the consideration set literature utilizes a two-stage framework: In the first stage, the decision maker forms the consideration set, i.e., a subset of the set of all available options. In the second stage, the decision maker chooses one element from the consideration set. In our model, consumers can only buy from firms inside their attention radius. The literature usually assumes that inside the consideration set the decision is made rationally (e.g., Manzini and Mariotti, 2018). We adhere to this assumption. Yet, the literature differs on the

[1]This approach can also be generalized to demand functions independent of the Hotelling real line and by further assuming that a fraction of consumers are aware of only one firm (Cosandier, Garcia, and Knauff, 2018).

formation of the consideration set: Eliaz and Spiegler (2011a,b), for example, assume that the formation is deterministic, whereas, for example, Manzini and Mariotti (2018) assume that it is probabilistic.

Eliaz and Spiegler (2011a,b) assume that in a market with two firms, consumers are only aware of their default firm. This default firm is firm 1 for half of the consumers and firm 2 for the other half of the consumers. This allocation is random. Firms produce goods or menus of goods and can induce the rival's consumers to consider them via marketing strategies (Eliaz and Spiegler, 2011a) or via producing attention grabbers (Eliaz and Spiegler, 2011b). The formation criteria of Eliaz and Spiegler (2011b) also utilize similarity. The authors discuss the case that attention grabbers only grab attention if they are similar to the rival's menu. In contrast, in our model similarity to the consumer's taste is the driving factor behind attention. Both models (Eliaz and Spiegler, 2011a,b) abstract from price setting and consider only homogeneous consumer preferences. In addition, allocation of consumers to the default is random. Eliaz and Spiegler (2011a,b) find that profits are the same as with fully rational consumers, but that consumers are worse off.

Manzini and Mariotti (2018) also discuss similarity as a formation criteria. Nevertheless, Manzini and Mariotti (2018) assume that an option makes it into the consideration set of the decision maker probabilistically: The higher the salience of the option, the more likely that the option enters the consideration set. Options can invest in their salience to increase this probability. Salience, for example, means standing out. Being similar to other options in the choice set thus decreases salience. As in Eliaz and Spiegler (2011a), similarity is measured against the other available options, whereas in our model, similarity is measured against the consumer's preferences. Specifically, we assume that consumers only notice that a particular good of a firm exists if that good is inside the consumers' attention radius.[2] One way to interpret this assumption is by assuming—as Manzini, Mariotti, and Tyson (2013) do in

[2]This attention radius implies that the firms may be unable to reach the whole market. The literature discusses similar constraints besides limited attention. For instance, Cancian, Bills, and Bergstrom (1995) assume that firms can only sell to consumers who are located on one side of them.

their choice-theoretic model—that a threshold exists.

The attention radii of consumers thus suggest that each firm potentially faces two groups of consumers: One group notices only one firm, the other group notices both firms. We assume that firms can distinguish between those groups. The firms are thus able to offer the good at different prices to the two groups. Price discrimination between informed and uninformed consumers is also, for example, discussed in Heidhues and Köszegi (2017) and Armstrong and Vickers (2018). Yet, in contrast to our model, Heidhues and Köszegi (2017) and Armstrong and Vickers (2018) focus on a distinction of informed and uninformed consumers that is independent of the consumers' preferences. Generally, the literature on price discrimination in spatial models is very broad and includes price differentiation with respect to valuation, preference for differentiation, or location of consumers (see, e.g., Armstrong, 2006, for a survey).

3 The Model

We consider a market for a horizontally differentiated product where two firms, firm 1 and firm 2, compete for a unit mass of consumers. We assume that firms have identical marginal costs that we set to 0. The consumers are uniformly distributed on the interval $[0, 1]$. The location $x \in [0, 1]$ of a consumer describes the consumer's most preferred version of the good. Initially, firms decide which version of the good to produce by choosing their positions $y_1, y_2 \in [0, 1]$. Without loss of generality, we assume $y_1 \leq y_2$.

Each consumer wants to buy exactly one unit of the good. If a consumer does not buy the good, her utility is normalized to 0. If the consumer located at $x \in [0, 1]$ buys the good from firm $i \in \{1, 2\}$, the consumer's utility is

$$u_x(i) = v - p_i - (x - y_i)^2,$$

where p_i is the price at which firm i sells the good, $y_i \in [0, 1]$ is the location of firm i, and

v is the gross utility of the good. We assume $v > 3$; this ensures that, in equilibrium, all consumers who notice at least one firm buy from one of these firms.

However, in our model, consumers' attention is limited and this constraint may prevent purchase: Each consumer only considers firms within her *attention radius* κ. The consumer at position x, then, only notices firm i on position y_i if $|x - y_i| \leq \kappa$, where $0 < \kappa \leq 1$. Firms thus make it into the consideration set[3] of a consumer, if they produce a version of the good that fits the consumer's taste well enough. If $|x - y_i| > \kappa$, the consumer does not even know (or remember) that firm i exists and, consequently, does not consider buying from firm i. Thus limited attention may prevent purchase from a firm that, potentially, has the overall better offer. Generally, if $\kappa = 1$, every consumer on $[0, 1]$ observes any point in $[0, 1]$. Therefore, this limiting case represents the standard Hotelling model where the choice set is identical to the consideration set.

From the perspective of firm $i \in \{1, 2\}$, the attention radii of consumers suggest that the firm can only reach consumers who are close enough. That means, the firm can only reach consumers that are inside its *radius of attentive consumers*, i.e., within the interval $[y_i - \kappa, y_i + \kappa]$. Consumers outside the radius of attentive consumers of firm i do not perceive firm i and thus never buy from firm i. Thus consumers' limited attention restricts the demand firms can capture. To derive the demand of the firms, we have to distinguish two cases: Either the radii of attentive consumers of firm 1 and firm 2 overlap or do not overlap.

If the firms' radii do not overlap, i.e., $[y_1 - \kappa, y_1 + \kappa] \cap [y_2 - \kappa, y_2 + \kappa] = \emptyset$ or, equivalently, $y_1 + \kappa \leq y_2 - \kappa$, no consumer who notices firm 1 notices firm 2 and vice versa. Thus each firm is a monopolist in its radius. All consumers $x \in [y_1 - \kappa, y_1 + \kappa]$ have a utility $v - p_1 - (x - y_1)^2 \geq 0$ and buy from firm 1; everyone else does not buy from firm 1. Then,

[3] The consideration set is a subset of the choice set. The choice set includes all available options (here, buying from firm 1 or firm 2, or not buying). The consideration set includes only those elements the consumer actively considers (here, not buying and buying from any of the firms inside the consumer's attention radius).

firm 1's demand is

$$x_1^m = y_1 + \kappa - \max\{0, y_1 - \kappa\}.$$

Similarly, for firm 2

$$x_2^m = \min\{y_2 + \kappa, 1\} - (y_2 - \kappa).$$

Figure 2 illustrates such a situation for $y_1 > \kappa$ and $y_2 > 1 - \kappa$: Then, the demand of firm 1 is $x_1^m = y_1 + \kappa - (y_1 - \kappa)$ and the demand of firm 2 is $x_2^m = 1 - (y_2 - \kappa)$.

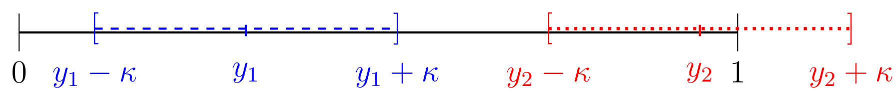

Figure 2: Example of non-overlapping radii of attentive consumers of firm 1 (blue/dashed) and firm 2 (red/dotted).

If the radii overlap, i.e., $[y_1 - \kappa, y_1 + \kappa] \cap [y_2 - \kappa, y_2 + \kappa] \neq \emptyset$ or, equivalently, $y_1 + \kappa > y_2 - \kappa$, some consumers notice both firms (see figure 3 for an example). In particular, all consumers $x \in [0, 1]$ such that $y_1 - \kappa \leq x < y_2 - \kappa$ notice only firm 1. All consumers $x \in [0, 1]$ such that $y_2 - \kappa \leq x \leq y_1 + \kappa$ notice both firms. All consumers $x \in [0, 1]$ such that $y_1 + \kappa < x \leq y_2 + \kappa$ notice only firm 2. Consumers buy from firm 1 if they see only firm 1 or see both firms and prefer firm 1, i.e., $v - p_1 - (x - y_1)^2 \geq v - p_2 - (x - y_2)^2$. Similarly, consumers buy from firm 2 if they see only firm 2 or see both firms and prefer firm 2. We denote the consumer who is indifferent between buying from firm 1 and buying from firm 2 by

$$\hat{x} = \frac{p_2 - p_1}{2(y_2 - y_1)} + \frac{y_1 + y_2}{2}. \tag{1}$$

In the following, we analyze how consumers' limited attention influences market outcomes. First, in section 4, we discuss the effects of limited attention on product differentiation if the price is exogenously fixed at some price p. Second, in section 5, we discuss the effects of limited attention if prices are endogenously set by the non-cooperative firms.

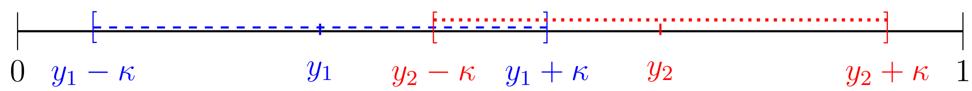

Figure 3: Example of overlapping radii of attentive consumers of firm 1 (blue/dashed) and firm 2 (red/dotted).

4 Exogenous Price

In this section, we analyze the direct effects of consumers' limited attention on product differentiation if prices are exogenously fixed such that $p_i = p$ for all $i \in \{1, 2\}$. We assume $0 < p \leq v - \kappa^2$. This assumption ensures that all consumers who notice at least one firm are willing to buy from one of these firms. As prices are identical, the indifferent consumer (1) is given by

$$\hat{x} = \frac{y_1 + y_2}{2}.$$

Firms play a one-stage game in which they choose their location in the product space to maximize their profits. Firm 1's demand is

$$x_1^{FP}(y_1, y_2) = \min\{y_1 + \kappa, \hat{x}\} - \max\{0, y_1 - \kappa\}.$$

Similarly, for firm 2

$$x_2^{FP}(y_1, y_2) = \min\{y_2 + \kappa, 1\} - \max\{\hat{x}, y_2 - \kappa\}.$$

With marginal costs set to 0, the profit of firm 1 is

$$\Pi_1^{FP}(y_1, y_2) = p\ x_1^{FP}$$

and the profit of firm 2 is

$$\Pi_2^{FP}(y_1, y_2) = p\ x_2^{FP}.$$

Firms choose their locations to maximize profits. Proposition 1 characterizes the equilibrium locations of firm 1 and firm 2 dependent on κ.

Proposition 1 *Characterization of the Nash equilibria in the model with exogenous prices dependent on the attention radius* κ*:*

(i) For $0 < \kappa \leq 1/4$*, any pair of locations* (y_1^*, y_2^*) *is an equilibrium if and only if* $y_1^* \in [\kappa, 1 - 3\kappa]$ *and* $y_2^* \in [y_1^* + 2\kappa, 1 - \kappa]$*. In any equilibrium, the profits are* $\Pi_1^* = \Pi_2^* = 2\kappa p$*.*

(ii) For $1/4 < \kappa \leq 1/2$*, the unique equilibrium locations are* $(y_1^* = \kappa, y_2^* = 1 - \kappa)$*. The equilibrium profits are* $\Pi_1^* = \Pi_2^* = p/2$*.*

(iii) For $\kappa > 1/2$*, the unique equilibrium locations are* $(y_1^* = 1/2, y_2^* = 1/2)$*. The equilibrium profits are* $\Pi_1^* = \Pi_2^* = p/2$*.*

Figure 4 illustrates the equilibrium locations for different values of κ. For $\kappa < 1/4$, a continuum of equilibrium locations exists. The gray area illustrates the locations of firm 1 and firm 2. For $\kappa \geq 1/4$, the equilibrium locations are unique.

Firms never choose locations such that their radii of attentive consumers overshoot the interval $[0, 1]$. Therefore, firm 1 never chooses a location $y_1 < \kappa$ and firm 2 never chooses a location $y_2 > 1 - \kappa$. Furthermore, both firms want to avoid an overlap of their radii of attentive consumers. As long as $\kappa \leq 1/4$, firms are able to choose locations to avoid an overlap. For $\kappa < 1/4$, a range of such locations exists. When $\kappa > 1/4$, firms are not able to avoid an overlap but choose locations that reduce the extent of the overlap. Firm 1, therefore, never chooses a location $y_1 > \kappa$ and firm 2 never chooses a location $y_2 > 1 - \kappa$ as long as $1/4 < \kappa \leq 1/2$. When $\kappa > 1/2$, both firms choose the median position to ensure that their

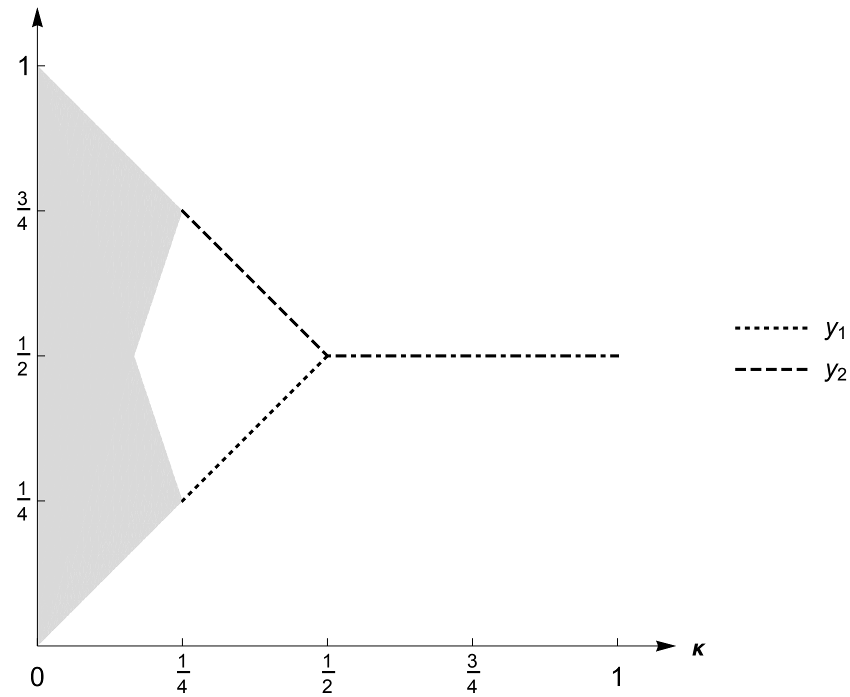

Figure 4: Equilibrium locations of firm 1 (dotted) and firm 2 (dashed) as a function of κ. For $\kappa \leq 1/4$ a continuum of equilibria exists, which is illustrated by the gray area.

radii of attentive consumers cover the entire product space $[0, 1]$ and the market is equally split among the firms. See appendix A.1 for a complete proof.

For $0 < \kappa \leq 1/4$, firms locate such that no consumer notices both firms. Then, the consumers in $[y_1 - \kappa, y_1 + \kappa]$ buy from firm 1, the consumers in $[y_2 - \kappa, y_2 + \kappa]$ buy from firm 2, and some consumers notice neither firm and are unable to buy the good. Thus the consumer surplus is

$$CS = \int_{y_1-\kappa}^{y_1+\kappa} v - p - (x - y_1)^2 dx + \int_{y_2-\kappa}^{y_2+\kappa} v - p - (x - y_2)^2 dx = 4\kappa(v - p - \frac{\kappa^2}{3})$$

and the producer surplus is

$$PS = \Pi_1^* + \Pi_2^* = 4\kappa p.$$

For $\kappa < 1/4$, as some consumers notice neither firm, those consumers do not participate in the market. As κ increases, the number of consumers who notice neither firm decreases. Consequently, the consumer surplus and the producer surplus, and thus the overall welfare, are increasing in κ as long as $0 < \kappa \leq 1/4$.

For $1/4 < \kappa \leq 1/2$, all consumers notice at least one firm and thus buy a good. In equilibrium, firms locate such that they split the market equally. The consumer surplus becomes

$$CS = \int_0^{1/2} v - p - (x-\kappa)^2 dx + \int_{1/2}^{1} v - p - (x-(1-\kappa))^2 dx = v - p - \kappa^2 + \frac{\kappa}{2} - \frac{1}{12}$$

and the producer surplus becomes

$$PS = \Pi_1^* + \Pi_2^* = p.$$

Because the locations are such that each firm always captures half of the consumers and prices are fixed, firms have no possibility to further increase their profits. Therefore, producer surplus is constant in κ. As κ increases, in equilibrium, firms choose to locate closer to the median consumer and thus increase the mean distance between consumers' and firms' locations. Consequently, consumer surplus decreases in κ. As producer surplus is constant and consumer surplus is decreasing in κ, welfare decreases in κ.

For $\kappa > 1/2$, the consumer surplus is

$$CS = \int_0^{1/2} v - p - \left(x - \frac{1}{2}\right)^2 dx + \int_{1/2}^{1} v - p - \left(x - \frac{1}{2}\right)^2 dx = v - p - \frac{1}{12}$$

and the producer surplus is

$$PS = \Pi_1^* + \Pi_2^* = p.$$

As long as $\kappa > 1/2$, the equilibrium locations are fixed at the location of the median consumer $y_1^* = y_2^* = 1/2$. This corresponds to the standard Hotelling result (Tirole, 1988). Consumer and producer surplus and, therefore, welfare, are constant in κ. Proposition 2 summarizes the welfare analysis.

Proposition 2 *Welfare analysis for exogenous prices:*

(i) Consumer surplus reaches its maximum at $\kappa = 1/4$*.*

(ii) Producer surplus reaches its maximum for all $\kappa \in [1/4, 1]$*.*

(iii) Welfare reaches its maximum at $\kappa = 1/4$*.*

Proposition 2 shows that the highest welfare level is achieved at $\kappa = 1/4$, where all consumers notice exactly one firm and participate in the market and the average distance between consumers' and firms' locations is minimized (firms choose $y_1 = 1/4$ and $y_2 = 3/4$). At $\kappa = 1/4$, consumer surplus and producer surplus also reach their maxima. In the standard Hotelling model, which our model captures at $\kappa = 1$, all consumers always notice both firms. This increases competition and induces firms to locate at the median consumer. In contrast, in our model, limited attention of consumers weakens competition as the number of consumers who notice both firms and for which firms compete is constrained. For low levels of attention, firms have an incentive to differentiate their products to capture more consumers who otherwise would not participate in the market as they notice neither firm. Therefore, firms locate closer to the efficient locations $y_1 = 1/4$ and $y_2 = 3/4$ under limited attention. Thus under exogenously fixed prices some level of inattention is actually beneficial to consumers.

5 Endogenous Prices with Price Discrimination

In this section, we analyze the effects of limited attention on product differentiation when firms are also able to set prices. Then, the two firms play a two-stage game: In stage one, firms simultaneously and independently choose locations in the product space; in stage two, each firm observes the location of its competitor and, then, the firms simultaneously and independently set prices. Each firm (potentially) faces two groups of consumers. Consumers who notice one firm and consumers who notice both firms. Firms are monopolists for consumers who notice only one firm, but have to compete for the consumers who notice both firms. By choosing their location in the product space, firms can influence the size of their two groups of consumers. We assume that firms can distinguish between those two groups of consumers and thus charge different prices from the two groups. Then, firms charge a monopoly price p_i^m from the consumers who notice only one firm, and a competition price p_i^c from the consumers who notice both firms. We solve for subgame-perfect equilibria by backward induction.

In the price-setting stage, firms set prices to maximize profits given the locations chosen in the first stage. Profits can be split into two parts; the profits from the monopoly and the profits from competition:

$$\Pi_1(p_1^m, p_1^c, p_2^c, y_1, y_2) = \Pi_1^m(p_1^m, y_1, y_2) + \Pi_1^c(p_1^c, p_2^c, y_1, y_2)$$
$$\Pi_2(p_2^m, p_2^c, p_1^c, y_1, y_2) = \Pi_2^m(p_2^m, y_1, y_2) + \Pi_2^c(p_2^c, p_1^c, y_1, y_2).$$

As firms set two different prices, we can solve for the two prices separately. Firm 1's monopoly demand consists of all consumers who notice only firm 1, i.e., $x \in [y_1 - \kappa, y_1 + \kappa] \cap [0, 1]$ and $x \notin [y_2 - \kappa, y_2 + \kappa]$, and whose utility exceeds zero: $u_1(x) = v - p_1^m - (x - y_1)^2 \geq 0 \Leftrightarrow y_1 - \sqrt{v - p_1^m} \leq x \leq y_1 + \sqrt{v - p_1^m}$. Thus as long as $p_1^m \leq v - \kappa^2$, all consumers who notice only firm 1 have a positive utility and buy from firm 1. If $v > p_1^m > v - \kappa^2$, all consumers who notice only firm 1 and are in $[y_1 - \sqrt{v - p_1^m}, y_1 + \sqrt{v - p_1^m}]$ have a positive utility and buy from firm 1. If $p_1^m > v$, the monopoly price exceeds the gross utility of all consumers and no consumer buys from firm 1. Thus the profit of firm 1 from the monopoly is

$$\Pi_1^m(p_1^m, y_1, y_2) = p_1^m \begin{cases} (\min\{y_2 - \kappa, y_1 + \kappa\} - \max\{0, y_1 - \kappa\}) & \text{if } p_1^m \leq v - \kappa^2 \\ (\min\{y_2 - \kappa, y_1 + \sqrt{v - p_1^m}\} - \max\{0, y_1 - \sqrt{v - p_1^m}\}) & \text{if } v - \kappa^2 < p_1^m \leq v \\ 0 & \text{if } v < p_1^m. \end{cases}$$

Similarly, the profit of firm 2 from the monopoly is

$$\Pi_2^m(p_2^m, y_1, y_2) = p_2^m \begin{cases} (\min\{y_2+\kappa, 1\} - \max\{y_2-\kappa, y_1+\kappa\}) & \text{if } p_2^m \leq v-\kappa^2 \\ (\min\{y_2+\sqrt{v-p_2^m}, 1\} - \max\{y_2-\sqrt{v-p_2^m}, y_1+\kappa\}) & \text{if } v-\kappa^2 < p_2^m \leq v \\ 0 & \text{if } v < p_2^m. \end{cases}$$

In general, the maximum monopoly demand that firm 1 can receive is given by $\min\{y_2-\kappa, y_1+\kappa\} - \max\{0, y_1-\kappa\}$. For $v > 3$, firm 1 has an incentive to set its monopoly price such that all consumers who notice only firm 1 are willing to buy from firm 1. The detailed derivation is in appendix A.2.

If the firms' radii of attentive consumers do not overlap (i.e., $y_1+\kappa \leq y_2-\kappa$), the last consumer who notices only firm 1 is at $x = y_1+\kappa$ and firm 1 sets a price $p_1^m = v-\kappa^2$. If the firms' radii overlap (i.e., $y_1+\kappa > y_2-\kappa$) and $y_1 \geq \kappa$, the last consumer who notices only firm 1 is at $x = y_1-\kappa$ and firm 1 sets a price $p_1^m = v-\kappa^2$. Thus when firm 1 can fully exploit one side of its radius, firm 1 sets the monopoly price such that all of these consumers are willing to buy from firm 1. Otherwise, firm 1 sets its monopoly price to capture the last consumer who notices only firm 1. Then, if the radius of firm 1 yields more monopoly demand on the left side than on the right side of firm 1 (i.e., $y_1 - 0 \geq y_2-\kappa-y_1$), the last consumer who notices just firm 1 is at $x = 0$ and firm 1 sets a price $p_1^m = v - y_1^2$. If the radius yields more demand on the right side (i.e., $y_1 - 0 < y_2-\kappa-y_1$), the last consumer who notices just firm 1 is at $x = y_2-\kappa$ and firm 1 sets a price $p_1^m = v-(y_1-y_2+\kappa)^2$.

The monopoly price of firm 1 is, therefore,

$$p_1^{m*} = \begin{cases} v-\kappa^2 & \text{if } y_1+\kappa \leq y_2-\kappa \text{ or } y_1+\kappa > y_2-\kappa \text{ with } y_1 \geq \kappa \\ v-y_1^2 & \text{if } y_1+\kappa > y_2-\kappa \text{ with } y_2-\kappa-y_1 \leq y_1 < \kappa \\ v-(y_1-y_2+\kappa)^2 & \text{if } y_1+\kappa > y_2-\kappa \text{ with } y_1 < y_2-\kappa-y_1 \text{ with } y_1 < \kappa. \end{cases}$$

Similarly, the monopoly price of firm 2 is

$$p_2^{m*} = \begin{cases} v - \kappa^2 & \text{if } y_1 + \kappa \leq y_2 - \kappa \text{ or } y_1 + \kappa > y_2 - \kappa \text{ with } y_2 \leq 1 - \kappa \\ v - (1 - y_2)^2 & \text{if } y_1 + \kappa > y_2 - \kappa \text{ with } 1 - y_2 \geq y_2 - y_1 - \kappa \text{ with } y_2 > 1 - \kappa \\ v - (y_2 - y_1 - \kappa)^2 & \text{if } y_1 + \kappa > y_2 - \kappa \text{ with } 1 - y_2 < y_2 - y_1 - \kappa \text{ with } y_2 > 1 - \kappa. \end{cases}$$

If the firms' radii of attentive consumers overlap, firms also face consumers who notice both firms. That means, firms compete for consumers in the interval $[y_2 - \kappa, y_1 + \kappa] \cap [0, 1] = [\max\{0, y_2 - \kappa\}, \min\{y_1 + \kappa, 1\}]$. All consumers in this interval located to the left of the indifferent consumer $\hat{x}$ buy from firm 1, all others from firm 2. In equilibrium, firms set prices such that both firms receive some demand.[4] If $y_1 \neq y_2$, the competition profits of firm 1 and firm 2 are

$$\Pi_1^c(p_1^c, p_2^c, y_1, y_2) = p_1^c\left(\hat{x} - \max\{0, y_2 - \kappa\}\right) = p_1^c\left(\frac{p_2^c - p_1^c}{2(y_2 - y_1)} + \frac{y_1 + y_2}{2} - \max\{0, y_2 - \kappa\}\right)$$
$$\Pi_2^c(p_2^c, p_1^c, y_1, y_2) = p_2^c\left(\min\{y_1 + \kappa, 1\} - \hat{x}\right) = p_2^c\left(\min\{y_1 + \kappa, 1\} - \frac{p_2^c - p_1^c}{2(y_2 - y_1)} - \frac{y_1 + y_2}{2}\right).$$

Firms set their prices p_1^c and p_2^c to maximize profits. If $y_1 + \kappa > y_2 - \kappa$, the best replies of firm 1 and firm 2 are

$$p_1^{c*}(p_2^c) = \frac{p_2^c}{2} + (y_1 - y_2)\left(\max\{0, y_2 - \kappa\} - \frac{y_1 + y_2}{2}\right)$$
$$p_2^{c*}(p_1^c) = \frac{p_1^c}{2} + (y_1 - y_2)\left(-\min\{y_1 + \kappa, 1\} + \frac{y_1 + y_2}{2}\right).$$

[4]If both firms would set prices such that one firm receives the full competition demand and the other firm receives zero competition demand, the firm that receives zero demand can strictly increase its profit by choosing the (strictly positive) price of its competitor. Thus such prices cannot exist in equilibrium.

The equilibrium prices are, then,

$$
\begin{aligned}
p_1^{c*} &= \frac{1}{3}(y_1 - y_2)(4\max\{0, y_2 - \kappa\} - 2\min\{y_1 + \kappa, 1\} - y_1 - y_2) \\
p_2^{c*} &= \frac{1}{3}(y_1 - y_2)(2\max\{0, y_2 - \kappa\} - 4\min\{y_1 + \kappa, 1\} + y_1 + y_2).
\end{aligned}
$$

The prices are increasing in the distance between firm 1 and firm 2. If firms have chosen the same location in the first stage, i.e., $y_1 = y_2$, price competition will ensure that $p_1^{c*} = p_2^{c*} = 0$. Taking these equilibrium prices, p_1^{m*}, p_2^{m*}, p_1^{c*}, and p_2^{c*}, the updated profits are

$$
\Pi_1(y_1, y_2) = \begin{cases} p_1^{m*} x_1^m & \text{if } y_1 + \kappa \leq y_2 - \kappa \\ p_1^{m*} x_1^m + p_1^{c*} x_1^c & \text{if } 0 < y_2 - \kappa < y_1 + \kappa \\ p_1^{c*} x_1^c & \text{if } y_2 - \kappa \leq 0 < y_1 + \kappa \end{cases} \tag{2}
$$

$$
\Pi_2(y_1, y_2) = \begin{cases} p_2^{m*} x_2^m & \text{if } y_1 + \kappa \leq y_2 - \kappa \\ p_2^{m*} x_2^m + p_2^{c*} x_2^c & \text{if } y_2 - \kappa < y_1 + \kappa < 1 \\ p_2^{c*} x_2^c & \text{if } y_2 - \kappa < 1 \leq y_1 + \kappa \end{cases} \tag{3}
$$

$$
\begin{aligned}
\text{where} \quad x_1^m &= \min\{y_2 - \kappa, y_1 + \kappa\} - \max\{0, y_1 - \kappa\} \\
x_2^m &= \min\{y_2 + \kappa, 1\} - \max\{y_2 - \kappa, y_1 + \kappa\} \\
x_1^c(p_1^{c*}, p_2^{c*}) &= -\frac{1}{6}\left(4\max\{0, y_2 - \kappa\} - 2\min\{y_1 + \kappa, 1\} - y_1 - y_2\right) \\
x_2^c(p_1^{c*}, p_2^{c*}) &= -\frac{1}{6}\left(2\max\{0, y_2 - \kappa\} - 4\min\{y_1 + \kappa, 1\} + y_1 + y_2\right)
\end{aligned}
$$

In the first stage, firms maximize profits by choosing their location in the product space. The structure of the profit functions (2) and (3) gives rise to a multitude of case distinctions. The first case of each profit function captures the situation that no consumer notices both firms. Thus both firms operate as pure monopolists. The second case captures the situation that firm i faces a subgroup of consumers who only notice firm i and a subgroup of consumers who also notice firm j. Therefore, the profit function consists of two terms: The profit from operating as a monopolist and the profit from competition. The third case captures that all consumers of firm i also notice firm j. Thus firm i only serves a competitive market. The size of the demand depends on the locations of the firms. Firms maximize profits over all cases to derive their best replies. Figure 5 illustrates the subgame-perfect equilibrium locations of firm 1 and firm 2.

If $0 < \kappa \leq 1/4$, the firms are able to choose locations such that both firms are monopolists in their complete radii of attentive consumers and firms will do so in all subgame-perfect equilibria. Therefore, firms radii of attentive consumers do not overlap. Assume the firms' locations induce an overlap of their radii, i.e., $y_1 + \kappa > y_2 - \kappa$. Then, for $0 < \kappa \leq 1/4$, either $y_1 > \kappa$, $y_2 < 1 - \kappa$, or both. This means, at least one firm is able to move farther away from the opponent and thereby gain additional monopoly demand by simultaneously losing competition demand. As the additional monopoly profit exceeds the lost competition profit, the firm will move farther away until it has reached a full monopoly. Then, if the other firm does not have a full monopoly, because its outer boundary overshoots the product range, e.g., $y_2 + \kappa > 1$, it will move closer to its opponent as it trades no demand for competition demand. This induces the other firm to move farther outwards again until both firms have full monopolies. Consequently, in all subgame-perfect equilibria, both firms have only monopoly demand and all pairs of locations that induce two full monopolies are subgame-perfect equilibria. See appendix A.3 for a formal proof.

If $\kappa > 1/4$, firms are unable to capture two full separate monopolies and competition becomes attractive for firms and is not avoided anymore. Nevertheless, as monopoly prices are higher than competition prices, firms prefer monopoly demand to competition demand. As κ increases, for fixed locations, more consumers notice both firms and the firms have to compete for these consumers. Generally, if the overlap of the radii of attentive consumers is small, few consumers notice both firms. For these consumers, the distance to the locations of both firms is about equally large. Therefore, for the choice of these consumers, the price is more relevant than the distance. Then, firms face price competition, which leads to lower competition prices. As the overlap increases, more consumers notice both firms. Therefore, the fraction of consumers for whom the distance is important for the consumption choice increases. This allows firms to extract higher surplus by setting higher prices. Nevertheless, competition prices are always lower than monopoly prices. Thus firms prefer to serve consumers as monopolists.

To dampen the effect that with increasing κ more consumers notice both firms, firms have an incentive to move outwards. Thus both firms only compete for a small number of consumers in the center of the product space and prefer to exploit as much monopoly rent as possible. However, as firms move outwards, a part of the radii of attentive consumers is outside $[0, 1]$. Thus the firms make no profit from $[y_1 - \kappa, 0)$ and $(1, y_2 + \kappa]$. When κ increases, these areas from which firms make no profits become larger and, despite firms moving outwards, more consumers notice both firms. As this also increases competition prices, competition becomes more tempting for firms. Finally, at $\kappa = (7-3\sqrt{3})/4$ competition is more attractive. Thus with increasing κ, firms move inwards to steal the business of their competitor and to receive a larger share of the competitive market.

As κ increases further, the competition demand increases as well and locating close to the center increases price competition among the firms. This reduces profits. Therefore, for $\kappa \geq (3\sqrt{3} - 4)/2$, firms move outwards to avoid competition which increases profits due to higher competition prices. At $\kappa = 3/4$, all consumers notice both firms, which means that the monopoly profit disappears. Nevertheless, as long as consumers are not fully attentive, not all consumers notice every part of $[0, 1]$. Thus firms have no incentive to directly locate at the extremes as this would enable the competitor to steal some fraction of the firm's demand and reduce its profits. In the limit as $\kappa = 1$, the classical Hotelling result of maximum product differentiation occurs. Figure 5 illustrates the subgame-perfect equilibrium locations of firm 1 and firm 2. See appendix A.3 for a formal proof.

Proposition 3 characterizes the subgame-perfect equilibria for all values of κ.

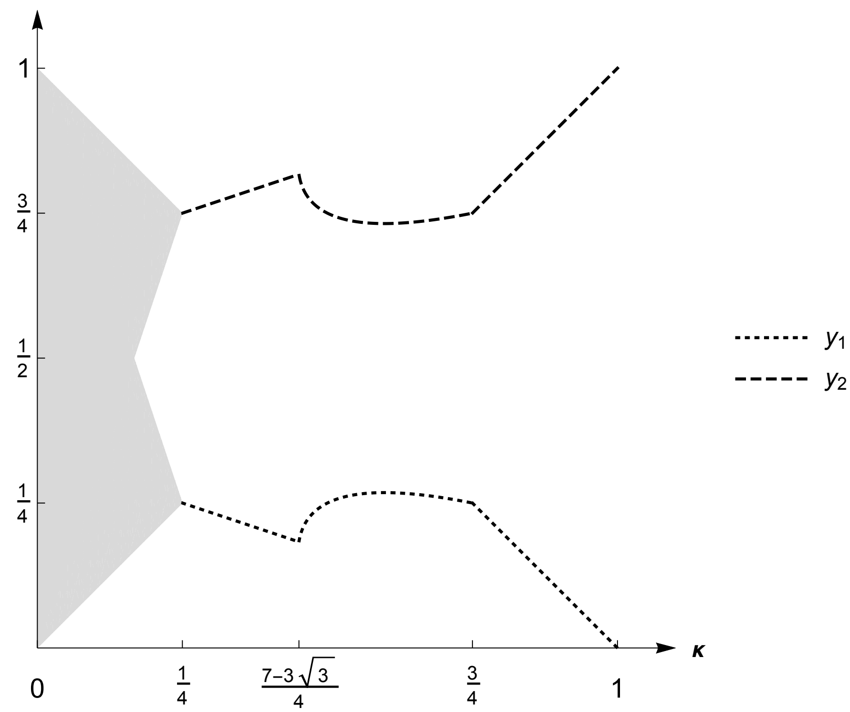

Figure 5: Subgame-perfect equilibrium locations of firm 1 (dotted) and firm 2 (dashed) as a function of κ. For $\kappa \leq 1/4$ a continuum of subgame-perfect equilibria exist, which is illustrated by the gray area .

Proposition 3 *Characterization of the subgame-perfect equilibria in the model with endogenous prices dependent on the attention radius κ:*

(i) If $0 < \kappa \leq 1/4$, any pair of locations (y_1^*, y_2^*) is a subgame-perfect equilibrium if and only if $y_1^* \in [\kappa, 1-3\kappa]$ and $y_2^* \in [y_2 + 2\kappa, 1-\kappa]$. The corresponding equilibrium prices are $p_1^{m*} = p_2^{m*} = v - \kappa^2$. In any subgame-perfect equilibrium, the profits are $\Pi_1^* = \Pi_2^* = (v - \kappa^2)2\kappa$.

(ii) If $1/4 < \kappa \leq (7-3\sqrt{3})/4$, the unique subgame-prefect equilibrium locations are $y_1^* = (1-\kappa)/3$ and $y_2^* = (2+\kappa)/3$. The corresponding equilibrium prices are $p_1^{m*} = p_2^{m*} = v - ((1-\kappa)/3)^2$ and $p_1^{c*} = p_2^{c*} = 1/9(1+2\kappa)(4\kappa-1)$. The profits are $\Pi_1^* = \Pi_2^* = p_1^{m*}(2-2\kappa)/3 - 1/6(1-4\kappa)p_1^{c*}$.

(iii) If $(7-3\sqrt{3})/4 < \kappa \leq 3/4$, the unique subgame-prefect equilibrium locations are $y_1^* = 1/4(2-3\kappa+\sqrt{\kappa^2+4\kappa-2})$ and $y_2^* = 1/4(2+3\kappa-\sqrt{\kappa^2+4\kappa-2})$. The cor-

responding equilibrium prices are $p_1^{m*} = p_2^{m*} = v - 1/16(2 - 3\kappa + \sqrt{\kappa^2 + 4\kappa - 2})^2$ *and* $p_1^{c*} = p_2^{c*} = 1/2(1 - 2\kappa + \kappa^2 + \kappa\sqrt{\kappa^2 + 4\kappa - 2})$. *The profits are* $\Pi_1^* = \Pi_2^* = 1/4(2 - \kappa - \sqrt{\kappa^2 + 4\kappa - 2})p_1^{m*} + 1/16(3\kappa - \sqrt{\kappa^2 + 4\kappa - 2})(\kappa + \sqrt{\kappa^2 + 4\kappa - 2})^2$.

(iv) If $\kappa > 3/4$, *the unique subgame-perfect equilibrium locations are* $y_1^* = 1 - \kappa$ *and* $y_2^* = \kappa$. *The corresponding equilibrium prices are* $p_1^{c*} = p_2^{c*} = 2\kappa - 1$. *The profits are* $\Pi_1^* = \Pi_2^* = (2\kappa - 1)/2$.

Proposition 3 shows that for $0 < \kappa \leq 1/4$, the subgame-perfect equilibrium locations are equivalent to the fixed price case (see proposition 1). But if firms face exogenous prices (see section 4), they tend towards the median location as κ increases. In contrast, if prices are endogenous, as $\kappa \to 1$, we approach maximum product differentiation ($y_1 \to 0$ and $y_2 \to 1$). Our model thus captures the standard result of d'Aspremont, Gabszewicz, and Thisse (1979)[5] as the limiting case of fully attentive consumers. For $\kappa = 1/4$, $\kappa = 1/2$, and $\kappa = 3/4$, firms choose the efficient locations, i.e., the locations that minimize the average distance between consumers' and firms' locations $y_1 = 1/4$ and $y_2 = 3/4$. Figure 6 illustrates the consumer surplus, the producer surplus, and the overall welfare for different levels of κ.

For $\kappa \leq 1/4$, in the subgame-perfect equilibrium, firms choose locations such that all consumers notice at most one firm. Thus both firms serve the market as monopolists. For $\kappa < 1/4$, some consumers notice neither firm and do not participate in the market. All consumers who notice a firm have to pay the monopoly price. The consumer surplus is, then,

$$CS = \int_{y_1-\kappa}^{y_1+\kappa} v - (v - \kappa^2) - (x - y_1)^2 dx + \int_{y_2-\kappa}^{y_2+\kappa} v - (v - \kappa^2) - (x - y_2)^2 dx = \frac{8}{3}\kappa^3$$

and the producer surplus is

$$PS = \Pi_1^* + \Pi_2^* = (v - \kappa^2)4\kappa.$$

[5] d'Aspremont, Gabszewicz, and Thisse (1979) analyze a Hotelling model where firms choose locations and prices and firms have quadratic transportation costs. They find, that firms maximally differentiate their products.

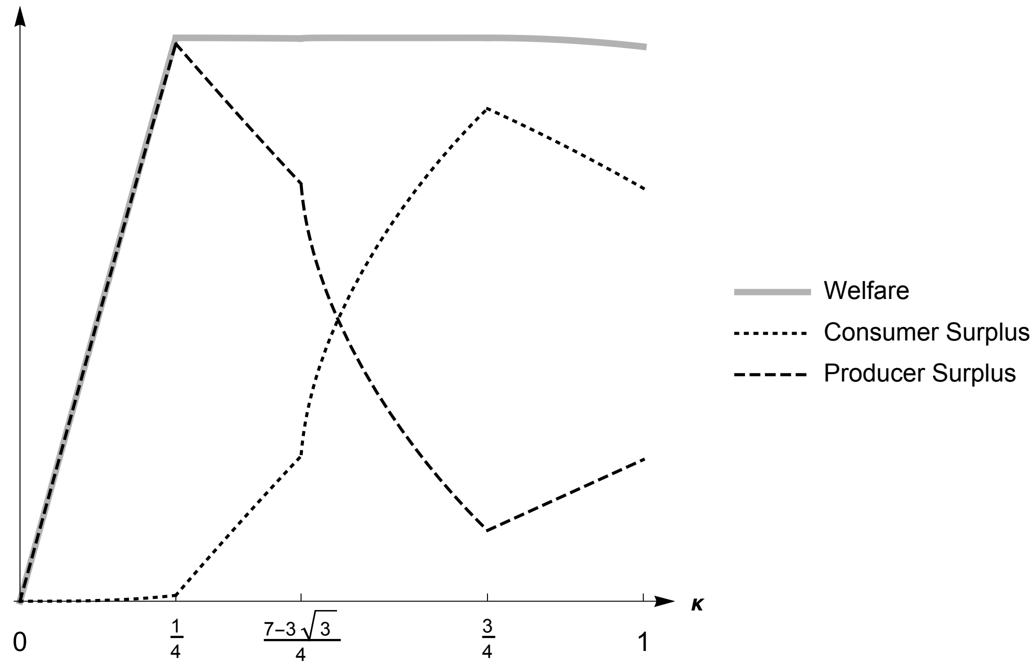

Figure 6: Welfare (solid), consumer surplus (dotted), and producer surplus (dashed) as a function of κ for $v = 4$.

As long as $\kappa \leq 1/4$, an increase in κ implies that firms can reach more consumers without facing competition. In addition, the fraction of consumers who do not participate in the market decreases. Consequently, consumer surplus, producer surplus, and welfare are increasing in κ. As all consumers pay the same (monopoly) price, the logic is similar to section 4.

For $\kappa > 1/4$, all consumers buy the good in the subgame-perfect equilibrium. Thus equilibrium prices are only relevant for the division of surplus between firms and consumers, but are irrelevant for total welfare. Welfare is only affected by equilibrium locations and the corresponding disutility consumers receive from buying a non-ideal version of the good. For $1/4 < \kappa \leq (7-3\sqrt{3})/4$, in the subgame-perfect equilibrium, firms choose locations such that all consumers notice at least one firm. Therefore, all consumers participate in the market. Some consumers notice only one firm and have to pay the monopoly price, whereas, the other consumers notice both firms and pay a lower competition price. Thus the consumer surplus is

$$
\begin{aligned}
CS = & \int_0^{y_2^*-\kappa} v - \left(v-(y_1^*)^2\right) - (x-y_1^*)^2 dx + \int_{y_2^*-\kappa}^{\hat{x}} v - \frac{1}{9}(1+2\kappa)(4\kappa-1) - (x-y_1^*)^2 dx \\
& + \int_{\hat{x}}^{y_1^*+\kappa} v - \frac{1}{9}(1+2\kappa)(4\kappa-1) - (x-y_2^*)^2 dx + \int_{y_1^*+\kappa}^{1} v - \left(v-(1-y_2^*)^2\right) - (x-y_2^*)^2 dx \\
= & v\frac{4\kappa-1}{3} - \frac{4}{3}\kappa^3 + \frac{1}{3}\kappa^2 - \frac{1}{6}\kappa + \frac{1}{12}
\end{aligned}
$$

and the producer surplus is

$$
PS = \Pi_1^* + \Pi_2^* = 2\left(\left(v - \left(\frac{1-\kappa}{3}\right)^2\right)\frac{2-2\kappa}{3} + \frac{(1+2\kappa)(1-4\kappa)^2}{54}\right).
$$

As κ increases, more consumers notice both firms, such that more consumers pay the lower competition price. Consequently, producer surplus is decreasing and consumer surplus is increasing in κ. In total, overall welfare is decreasing.

For $(7-3\sqrt{3})/4 < \kappa \leq 3/4$, the consumer surplus is

$$
\begin{aligned}
CS = & \int_0^{y_2^*-\kappa} v - \left(v-(y_1^*)^2\right) - (x-y_1^*)^2 dx + \int_{y_2^*-\kappa}^{\hat{x}} v - p_1^{c*} - (x-y_1^*)^2 dx \\
& + \int_{\hat{x}}^{y_1^*+\kappa} v - p_2^{c*} - (x-y_2^*)^2 dx + \int_{y_1^*+\kappa}^{1} v - \left(v-(1-y_2^*)^2\right) - (x-y_2^*)^2 dx \\
= & \frac{1}{48}\left(24v\kappa - 30\kappa^3 + 18\kappa^2 - 51\kappa + 20 + \sqrt{\kappa^2+4\kappa-2}(24v - 30\kappa^2 + 30\kappa - 9)\right)
\end{aligned}
$$

and the producer surplus is

$$
\begin{aligned}
PS = \Pi_1^* + \Pi_2^* = & \frac{1}{2}\left(v - \frac{1}{16}\left(2-3\kappa+\sqrt{\kappa^2+4\kappa-2}\right)^2\right)\left(2-\kappa-\sqrt{\kappa^2+4\kappa-2}\right) \\
& + \frac{1}{8}\left(3\kappa - \sqrt{\kappa^2+4\kappa-2}\right)\left(\kappa+\sqrt{\kappa^2+4\kappa-2}\right)^2.
\end{aligned}
$$

For $(7-3\sqrt{3})/4 < \kappa \leq 3/4$, in the subgame-perfect equilibrium, the distance between the locations of firm 1 and firm 2 decreases for $\kappa < (3\sqrt{3}-4)/2$ and increases for $\kappa > (3\sqrt{3}-4)/2$. The locations approach the efficient locations ($y_1 = 1/4$ and $y_2 = 3/4$) as $\kappa \to 1/2$ and

$\kappa \to 3/4$. This is beneficial to consumers and increases consumer surplus. Yet, increasing product differentiation decreases competition between firms and thus increases competition prices which reduces consumer surplus. However, as κ increases, more consumers notice both firms and more consumers pay the lower competition price. Overall, consumer surplus is increasing in κ. Firms exchange monopoly demand for competition demand. Overall therefore, producer surplus is decreasing in κ. Between $(7-3\sqrt{3})/4 < \kappa \leq 3/4$ welfare is reallocated from firms to consumers. In addition, at $\kappa = 1/2$ and at $\kappa = 3/4$ the firms choose locations that minimize the mean distance between consumers' and firms' locations. Therefore, the overall welfare reaches its maximum at $\kappa = 1/2$ and at $\kappa = 3/4$.

For $\kappa > 3/4$, firms locate such that all consumers see both firms and as κ increases $y_1 \to 0$ and $y_2 \to 1$. The consumer and the producer surplus are

$$\begin{aligned}
CS &= \int_0^{\hat{x}} v - (2\kappa - 1) - (x - (1-\kappa))^2 dx + \int_{\hat{x}}^1 v - (2\kappa - 1) - (x - \kappa)^2 dx \\
&= v - \kappa^2 - \frac{1}{2}\kappa + \frac{5}{12} \\
PS &= \Pi_1^* + \Pi_2^* = 2\kappa - 1.
\end{aligned}$$

For $\kappa > 3/4$, in the subgame-perfect equilibrium, the distance between the firms increases in κ, which allows firms to increase prices. This harms consumers and benefits firms. Therefore, consumer surplus is decreasing and producer surplus is increasing in κ. The overall welfare is decreasing.

Proposition 4 summarizes the welfare analysis.

Proposition 4 *Welfare analysis for endogenous prices:*

(i) Producer surplus reaches its maximum at $\kappa = 1/4$.

(ii) Consumer surplus reaches its maximum at $\kappa = 3/4$.

(iii) Welfare reaches its maximum at $\kappa = 1/4$, $\kappa = 1/2$, and $\kappa = 3/4$.

In summary, some degree of inattention is actually beneficial to consumers, because the consumers' inattention induces firms to decrease the average distance between consumers' and firms' location. In addition, limited attention also influences the prices consumers have to pay. The smaller κ, the more consumers have to pay the monopoly price instead of the lower competition price. Producer surplus is maximized at $\kappa = 1/4$, where the firms operate as independent monopolists; each firm for exactly half of the consumers. Thus firms cannot increase demand and sell at the monopoly price to all consumers. At $\kappa = 1/4$ consumers actually benefit from product differentiation as firms choose locations $y_1 = 1/4$ and $y_2 = 3/4$ which minimize the mean distance between consumers' and firms' locations. However, all consumers have to pay the monopoly price. Consumer surplus is maximized at $\kappa = 3/4$, where firms also locate at $y_1 = 1/4$ and $y_2 = 3/4$, but all consumers pay the lower competition price. In addition, the competition price is lower at $\kappa = 3/4$ than under full attention.

Under full attention, firms maximally differentiate their products to increase their market power which allows them to set higher prices. Therefore, consumers benefit from limited attention as limited attention induces more efficient product differentiation that is favorable to consumers and reduces firms' market power. Consumer surplus is maximized under limited attention and not under full attention. Between $\kappa = 1/4$ and $\kappa = 3/4$, welfare is reallocated from firms to consumers as more consumers pay the lower competition price instead of the monopoly price. In addition, at $\kappa = 1/4$, $\kappa = 1/2$, and $\kappa = 3/4$ as firms choose the efficient locations consumer surplus increases. Therefore, the overall welfare reaches its maximum at $\kappa = 1/4$, $\kappa = 1/2$, and $\kappa = 3/4$. That is, welfare is higher under limited than under full attention.

6 Conclusion

In this article, we demonstrate the effects of limited attention on horizontal product differentiation and the implications for welfare. To capture the effects of limited attention, we develop a novel method to model limited attention: An attention radius for each consumer. This radius restricts the consumers' focus to the fraction of the product space that is close to the consumers' preferred version of the good. Therefore, limited attention reduces competition among firms and thus it might classically be expected that limited attention is harmful to the consumers. However, we find that limited attention is only harmful to consumers for very low levels of attention, but that an intermediate level of attention is actually beneficial to consumers. At low levels of attention, some consumers notice neither firm and are, therefore, unable to participate in the market. But as attention increases, all consumers notice at least one firm. Then, consumers benefit from limited attention, because limited attention induces firms to differentiate their products. Overall, we find that welfare is higher for some degrees of limited attention than under full attention.

We make a number of limiting assumptions. We assume price discrimination between fully and partially informed consumers to keep the model tractable. Future research might address the question, how robust our results are to other forms of price setting such as uniform pricing or other degrees of price discrimination. In addition, we assume homogeneous attention radii with a cut-off, where consumers abruptly turn from attentive to inattentive. From a psychological perspective, the size of the attention radii might differ among consumers. For example, experts might have a different attention radius than lay persons. Alternatively, a consumer might have a different attention radius when she is fully awake than when she is tired. Adding such heterogeneity might change the behavior of firms and thus yield additional insights. Furthermore, relaxing the assumption of an abrupt cut-off towards a smoother transition between attention and inattention can be a fruitful avenue for future research.

Additionally, we frame our model in terms of horizontal product differentiation. Nevertheless, our model can easily be applied to other contexts, for example, political or spatial competition. Another interesting extension might be to identify other areas where our attention radius can be applied. For instance it might prove interesting to analyze the effects of our attention radius in other models of horizontal or vertical product differentiation.

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A Appendix

A.1 Proof Proposition 1

For $0 < \kappa \leq 1/2$, both firms locate in the interval $[\kappa, 1-\kappa]$. If a firm deviates to a location outside $[\kappa, 1-\kappa]$, its radius of attentive consumers overshoots the product range and it loses demand without any gain.

(i) For $0 < \kappa \leq 1/4$, in equilibrium, firms' radii of attentive consumers never overlap. Suppose radii would overlap, i.e., $y_1 + \kappa > y_2 - \kappa$, then, $y_1 > \kappa$ and/or $y_2 < 1 - \kappa$. Then, at least one of the two firms can strictly increase its profits by moving closer to κ or $1 - \kappa$ respectively and trading competition demand for monopoly demand. For $y_1^* \in [\kappa, 1 - 3\kappa]$ and $y_2^* \in [y_1^* + 2\kappa, 1 - \kappa]$, the radii of attentive consumers do not overlap and both firms earn their highest possible profits. All of these locations are Nash equilibria.

(ii) For $1/4 < \kappa \leq 1/2$, as in equilibrium $y_1, y_2 \in [\kappa, 1-\kappa]$, the firms' radii always overlap. Within this range, firm 1 minimizes the overlap by setting $y_1 = \kappa$ and firm 2 minimizes the overlap by setting $y_2 = 1 - \kappa$. This maximizes each firms' profit and thus forms the unique Nash equilibrium.

(iii) For $\kappa > 1/2$, firms are able to choose locations that ensure that all consumers in the market are within their radii. Firms locate at the median consumer's position: $y_1 = y_2 = 1/2$. This is a Nash equilibrium as any deviation by $\varepsilon < 1/2$ lowers the demand by $|\varepsilon|/2$. Further, there is no other equilibrium. Each firm must receive at least half of the demand, otherwise it would relocate to the median location. Both firms receive half of the demand either if they choose symmetric locations with $y_1 < 1/2$ and $y_2 > 1/2$ (but then each firm would benefit from relocating to $1/2$) or if they choose the same location $y_1 = y_2 \neq 1/2$ (but then each firm has an incentive to move closer to $1/2$).

A.2 Derivation of the Monopoly Prices

Assume firm 1's monopoly demand on one side is larger than the monopoly demand on the other side. Without loss of generality, we assume that the left side is the larger side. The distance from firm 1's location to the right end of the monopoly area can be denoted as $y_2 - \kappa - y_1$ (as the right side is constrained by the radius of attentive consumers of firm 2). Note that this value can also be negative such that the monopoly area is only on the left side of the firm. We can define $d \in [0, \min\{y_1, y_2 - \kappa - y_1, \kappa\}]$ as the distance between the consumer who is indifferent between buying the good at the monopoly price from firm 1 and not buying. Then, we can express the monopoly price and the monopoly profit as a function of the distance d:[6] $p_1^m = v - d^2$ and

$$\Pi_1(d) = (d + y_2 - \kappa - y_1)(v - d^2).$$

The optimal distance is

$$d^* \equiv -\frac{y_2 - y_1 - \kappa}{3} + \frac{1}{3}\sqrt{(y_2 - y_1 - \kappa)^2 + 3v} = \arg\max_d \Pi_1(d).$$

We find that the profit of firm 1 is strictly increasing for $d \in [0, d^*)$. Then, firm 1 is always willing to exploit the whole monopoly range if

$$d^* \geq \kappa \Leftrightarrow v \geq \kappa^2 + 2\kappa(y_2 - y_1).$$

As $0 \leq y_2 - y_1 \leq 1$, $0 < \kappa \leq 1$ and $v > 3$, firm 1 always exploits the whole market. By symmetry, the same holds true for monopolies where the larger part of the monopoly is on the right side of firm 1.[7] Thus in the asymmetric case, the monopoly price is always set to fully exploit the monopoly demand. This must then also be true in the symmetric case (when

[6] $u_1(d) = v - p_1^m - d^2 = 0 \Leftrightarrow p_1^m = v - d^2$.

[7] If the right demand side of firm 1 is larger, the profit changes to $\Pi_1 = (d + y_1)(v - d^2)$. However, $d^* \geq \kappa$ and the firms are willing to exploit the whole monopoly market.

the monopoly demand on the left side is as large as the monopoly demand on the right side), as now by setting a higher price, the firm would not only loose demand on one but on both sides.

As we have shown, firms have an incentive to always exploit the full monopoly demand. The monopoly price of firm 1 is, therefore,

$$p_1^{m*} = \begin{cases} v - \kappa^2 & \text{if } y_1 + \kappa \leq y_2 - \kappa \text{ or } y_1 + \kappa > y_2 - \kappa \text{ with } y_1 \geq \kappa \\ v - y_1^2 & \text{if } y_1 + \kappa > y_2 - \kappa \text{ with } y_2 - \kappa - y_1 \leq y_1 < \kappa \\ v - (y_1 - y_2 + \kappa)^2 & \text{if } y_1 + \kappa > y_2 - \kappa \text{ with } y_1 < y_2 - \kappa - y_1 \text{ with } y_1 < \kappa. \end{cases}$$

By symmetry, firm 2 also always exploits its whole monopoly market. Thus the monopoly price of firm 2 is

$$p_2^{m*} = \begin{cases} v - \kappa^2 & \text{if } y_1 + \kappa \leq y_2 - \kappa \text{ or } y_1 + \kappa > y_2 - \kappa \text{ with } y_2 \leq 1 - \kappa \\ v - (1 - y_2)^2 & \text{if } y_1 + \kappa > y_2 - \kappa \text{ with } 1 - y_2 \geq y_2 - y_1 - \kappa \text{ with } y_2 > 1 - \kappa \\ v - (y_2 - y_1 - \kappa)^2 & \text{if } y_1 + \kappa > y_2 - \kappa \text{ with } 1 - y_2 < y_2 - y_1 - \kappa \text{ with } y_2 > 1 - \kappa. \end{cases}$$

A.3 Proof Proposition 3

Proof subgame-perfect equilibria $0 < \kappa \leq 1/4$

Assume $0 < \kappa \leq 1/4$. The proof proceeds in three steps: First, we show that any pair of locations (y_1, y_2) such that the firms' radii of attentive consumers overlap (i.e., $y_1+\kappa > y_2-\kappa$) cannot be a subgame-perfect equilibrium. Second, we show that in the subgame-perfect equilibrium firms do not choose locations such that $y_i < \kappa$ or $y_i > 1-\kappa$. Third, we show that the remaining pairs of locations (y_1, y_2) such that $y_1 \in [\kappa, 1-3\kappa]$ and $y_2 \in [y_1+2\kappa, 1-\kappa]$ are the locations in the subgame-perfect equilibria.

Step 1: Any pair of locations such that $y_1 + \kappa > y_2 - \kappa$ can never be a subgame-perfect equilibrium. Suppose $y_1 + \kappa > y_2 - \kappa$, then one firm has an incentive to move away from the opponent without overshooting $[0, 1]$, which increases that firm's profit. With $y_1+\kappa > y_2-\kappa$ and $0 < \kappa \leq 1/4$, either $y_1 > \kappa$, $y_2 < 1 - \kappa$, or both. Suppose $y_1 > \kappa$,

$$\Pi_1(y_1, y_2) = (v - \kappa^2)(y_2 - y_1) + \frac{1}{18}(y_2 - y_1)(3y_2 - 4\kappa - 2\min\{y_1 + \kappa, 1\} - y_1)^2$$

$$\frac{\partial \Pi_1(y_1, y_2)}{\partial y_1} = \begin{cases} \underbrace{-v - \kappa^2}_{<-3} + \underbrace{4\kappa(y_2 - y_1)}_{<8\kappa^2 \leq \frac{1}{2}} \underbrace{-\frac{3}{2}(y_2 - y_1)^2}_{\leq 0} < 0 & \text{if } y_1 + \kappa \leq 1 \\ \underbrace{-(v - \kappa^2)}_{<0} - \frac{1}{18} \underbrace{(3y_2 - y_1 - 4\kappa - 2)}_{\leq 0} \underbrace{(5y_2 - 3y_1 - 4\kappa - 2)}_{\leq 0} < 0 & \text{if } y_1 + \kappa > 1 \end{cases}$$

and, by symmetry, if $y_2 < 1 - \kappa$, $\partial \Pi_2(y_1, y_2)/\partial y_2 > 0$. Thus if the firms' radii of attentive consumers overlap, at least one of the two firms has an incentive to deviate until the distance between y_1 and y_2 is large enough such that $y_1 + \kappa \leq y_2 - \kappa$.

Step 2: Any pair of locations such that $y_i < \kappa$ can never be a subgame-perfect equilibrium. Suppose $y_1 < \kappa$, then a part of the attention radius of firm 1 lies outside $[0, 1]$. Thus firm 1 can profitable deviate to $y_1 = \kappa$ to increase its profit. This either strictly increases monopoly profit or weakly increases monopoly profit and strictly increases competition profit. Suppose $y_2 < \kappa$, the radii of attentive consumers would overlap, which is excluded in the first step of this proof. Thus neither firm chooses a location $y_i < \kappa$. By symmetry, neither firm chooses a location $y_i > 1 - \kappa$.

Step 3: All remaining pairs of locations (y_1, y_2) such that $y_1 \in [\kappa, 1-\kappa]$ and $y_2 \in [\kappa, 1-\kappa]$ with $y_1 + \kappa \leq y_2 - \kappa$ are subgame-perfect equilibria. With each of these pairs of locations, firms receive the highest possible profit $\Pi_1 = \Pi_2 = (v - \kappa^2)2\kappa$. Thus neither firm has an incentive to deviate.

Proof subgame-perfect equilibria $1/4 < \kappa \leq 1/2$

Assume $1/4 < \kappa \leq 1/2$. The proof proceeds in four steps: First, we show that any pair of locations (y_1, y_2) where the firms' radii of attentive consumers do not overlap (i.e., $y_1 + \kappa \leq y_2 - \kappa$) cannot be a subgame-perfect equilibrium. Second, we show that in the subgame-perfect equilibrium firms do not choose locations such that $y_1 > \kappa$ and/or $y_2 < 1-\kappa$. Third, we show that firm 1 never chooses a location $y_1 < (y_2 - \kappa)/2$ and firm 2 never chooses a location $y_2 > (1 + y_1 + \kappa)/2$. Fourth, we specify the best replies and the subgame-perfect equilibria.

Step 1: Any pair of locations such that $y_1 + \kappa \leq y_2 - \kappa$ can never be a subgame-perfect equilibrium. Suppose $y_1 + \kappa \leq y_2 - \kappa$, then a part of the radius of at least one firm lies outside $[0, 1]$. This firm can profitable deviate to increase its profit by forcing an overlap. This increases monopoly profit and competition profit. Therefore, firms always choose locations such that $y_1 + \kappa > y_2 - \kappa$.

Step 2: Firm 1 never chooses a location $y_1 > \kappa$. Suppose $y_1 > \kappa$,

$$\Pi_1(y_1, y_2) = (v - \kappa^2)(y_2 - y_1) + \frac{1}{18}(y_2 - y_1)(3y_2 - 4\kappa - 2\min\{y_1 + \kappa, 1\} - y_1)^2$$

$$\frac{\partial \Pi_1(y_1, y_2)}{\partial y_1} = \begin{cases} -\underbrace{v}_{>3}\underbrace{-\kappa^2 + 4\kappa(y_2 - y_1)}_{<8\kappa^2-\kappa^2<3}\underbrace{-\frac{3}{2}(y_2 - y_1)^2}_{\leq 0} < 0 & \text{if } y_1 + \kappa < 1 \\ \underbrace{-(v - \kappa^2)}_{<0} - \frac{1}{18}\underbrace{(3y_2 - y_1 - 4\kappa - 2)}_{<0}\underbrace{(5y_2 - 3y_1 - 4\kappa - 2)}_{<0} < 0 & \text{if } y_1 + \kappa \geq 1. \end{cases}$$

The first derivative is strictly negative and firm 1 always has an incentive to move to the left. Therefore, firm 1 never chooses a location $y_1 > \kappa$. By symmetry, firm 2 never chooses a location $y_2 < 1 - \kappa$. Consequently, a potential subgame-perfect equilibrium must involve $y_1 \leq \kappa$ and $y_2 \geq 1 - \kappa$.

Step 3: As $y_1 \leq \kappa$ and $y_2 \geq 1 - \kappa$ with $y_1 + \kappa > y_2 - \kappa$, both firms locate close to the boundaries of the product space but also compete for consumers who notice both firms in the center. Profits for both firms become

$$\Pi_1(y_1, y_2) = \frac{1}{2}(y_2 - y_1)(y_2 - y_1 - 2\kappa)^2 + (y_2 - \kappa) \begin{cases} (v - (y_2 - y_1 - \kappa)^2) & \text{if } y_1 < \frac{y_2 - \kappa}{2} \\ (v - y_1^2) & \text{if } y_1 \geq \frac{y_2 - \kappa}{2} \end{cases}$$

$$\Pi_2(y_1, y_2) = \frac{1}{2}(y_2 - y_1)(y_2 - y_1 - 2\kappa)^2 + (1 - y_1 - \kappa) \begin{cases} (v - (y_2 - y_1 - \kappa)^2) & \text{if } y_2 > \frac{1 + y_1 + \kappa}{2} \\ (v - (1 - y_2)^2) & \text{if } y_2 \leq \frac{1 + y_1 + \kappa}{2} \end{cases}$$

First, suppose firm 1 would choose a location $y_1 < (y_2 - \kappa)/2$. As

$$\frac{\partial \Pi_1(y_1, y_2)}{\partial y_1} = 2 \underbrace{(y_2 - y_1 - \kappa)}_{>0 \text{ as } y_1 < \frac{y_2 - \kappa}{2}} \underbrace{(y_2 - \kappa)}_{>0} - \underbrace{\frac{3}{2}(y_2 - y_1 - 2\kappa)}_{<0} \underbrace{\left(y_2 - y_1 - \frac{2}{3}\kappa\right)}_{>0 \text{ as } y_1 < \frac{y_2 - \kappa}{2}} > 0,$$

firm 1 always has an incentive to move inwards for $y_1 < (y_2 - \kappa)/2$. By symmetry, the same holds for firm 2 choosing $y_2 > (1 + y_1 + \kappa)/2$. Then, a potential subgame-perfect equilibrium must involve locations such that $y_1 \in [(y_2 - \kappa)/2, \kappa]$ and $y_2 \in [1 - \kappa, (1 + y_1 + \kappa)/2]$.

Step 4: Now, we derive the best replies of firm 1 and firm 2 with $y_1 \in [(y_2 - \kappa)/2, \kappa]$ and $y_2 \in [1 - \kappa, (1 + y_1 + \kappa)/2]$ and, subsequently, specify the subgame-perfect equilibria. The first derivative of the profit functions of both firms is

$$\begin{aligned}
\frac{\partial \Pi_1(y_1,y_2)}{\partial y_1} &= -2y_1(y_2-\kappa) - \frac{3}{2}(y_2-y_1-2\kappa)\left(y_2-y_1-\frac{2}{3}\kappa\right) = 0 \\
&\Leftrightarrow y_1(y_2) = \frac{1}{3}\left(y_2-2\kappa \pm 2\sqrt{-2y_2^2+5\kappa y_2-2\kappa^2}\right) \\
\frac{\partial \Pi_2(y_1,y_2)}{\partial y_2} &= 2(1-y_2)(1-y_1-\kappa) + \frac{3}{2}(y_2-y_1-2\kappa)\left(y_2-y_1-\frac{2}{3}\kappa\right) = 0 \\
&\Leftrightarrow y_2(y_1) = \frac{1}{3}\left(2+y_1+2\kappa \pm 2\sqrt{-2y_1^2+y_1(4-5\kappa)+\kappa(5-2\kappa)-2}\right).
\end{aligned}$$

Checking the second order condition, we find that the potential maxima are[8]

$$\begin{aligned}
y_1(y_2) &= \frac{1}{3}\left(y_2-2\kappa+2\sqrt{-2y_2^2+5\kappa y_2-2\kappa^2}\right) \\
y_2(y_1) &= \frac{1}{3}\left(2+y_1+2\kappa-2\sqrt{-2y_1^2+y_1(4-5\kappa)+\kappa(5-2\kappa)-2}\right).
\end{aligned}$$

Note that these potential maxima must fulfill the conditions $y_1 \in [(y_2-\kappa)/2, \kappa]$ and $y_2 \in [1-\kappa, (1+y_1+\kappa)/2]$ to be a best reply. For simplicity, let us first focus on the derivation of the best reply function for firm 1. Consequently, for $y_1 \in [(y_2-\kappa)/2, \kappa]$ we must have

$$\frac{y_2-\kappa}{2} \leq \frac{1}{3}\left(y_2-2\kappa+2\sqrt{-2y_2^2+5\kappa y_2-2\kappa^2}\right) \leq \kappa.$$

If the best reply lies outside the range, the firm chooses the boundary solution. Checking both conditions we find that

$$y_1(y_2) = \begin{cases} \frac{y_2-\kappa}{2} & \text{if } y_2 < \frac{13-4\sqrt{3}}{11}\kappa \\ \frac{y_2-2\kappa+2\sqrt{-2y_2^2+5\kappa y_2-2\kappa^2}}{3} & \text{if } \frac{13-4\sqrt{3}}{11}\kappa \leq y_2 \leq \frac{13+4\sqrt{3}}{11}\kappa \\ \frac{y_2-\kappa}{2} & \text{if } y_2 > \frac{13+4\sqrt{3}}{11}\kappa. \end{cases} \tag{4}$$

[8]For firm 1 the potential maximum only exists if $y_2 \leq 2\kappa$. Suppose $y_2 > 2\kappa$, then $\partial \Pi_1(y_1,y_2)/\partial y_1 < 0$ and firm 1 chooses $y_1 = (y_2-\kappa)/2$. For firm 2 the potential maximum only exists if $y_1 \geq 1-2\kappa$. Suppose $y_1 < 1-2\kappa$, then $\partial \Pi_2(y_1,y_2)/\partial y_2 > 0$ and firm 2 chooses $y_2 = (1+y_1+\kappa)/2$.

Next, we need to check whether the conditions of (4) satisfy $[1-\kappa,(1+y_1+\kappa)/2]$ or are partly outside. First, we check

$$\begin{aligned} \frac{13-4\sqrt{3}}{11}\kappa \leq 1-\kappa \leq \frac{13+4\sqrt{3}}{11}\kappa \\ \frac{11}{24+4\sqrt{3}} \leq \kappa \leq \frac{11}{24-4\sqrt{3}} \end{aligned} \tag{5}$$

Later, we check the conditions for $(1+y_1+\kappa)/2$, when we analyze whether potential subgame-perfect equilibria are in the range of the best reply function. Using (4) and (5), we can rewrite the best reply function of firm 1:

If $\kappa < \frac{11}{24+4\sqrt{3}}$,

$$y_1^*(y_2) = \frac{y_2-\kappa}{2}.$$

If $\frac{11}{24+4\sqrt{3}} \leq \kappa \leq \frac{1}{2}$,

$$y_1^*(y_2) = \begin{cases} \frac{y_2-2\kappa+2\sqrt{-2y_2^2+5\kappa y_2-2\kappa^2}}{3} & \text{if } y_2 \leq \frac{13+4\sqrt{3}}{11}\kappa \\ \frac{y_2-\kappa}{2} & \text{if } y_2 > \frac{13+4\sqrt{3}}{11}\kappa \end{cases}$$

Checking the same conditions for firm 2, if $\kappa < \frac{11}{24+4\sqrt{3}}$,

$$y_2^*(y_1) = \frac{1+y_1+\kappa}{2}$$

and if $\frac{11}{24+4\sqrt{3}} \leq \kappa \leq \frac{1}{2}$,

$$y_2^*(y_1) = \begin{cases} \frac{1+y_1+\kappa}{2} & \text{if } y_1 < \frac{11-13\kappa-\sqrt{48}\kappa}{11} \\ \frac{2+y_1+2\kappa-2\sqrt{-2y_1^2+y_1(4-5\kappa)+\kappa(5-2\kappa)-2}}{3} & \text{if } \frac{11-13\kappa-\sqrt{48}\kappa}{11} \leq y_1 \leq \kappa \end{cases}$$

The intersections of the best replies gives the subgame-perfect equilibria.

Thus if $1/4 < \kappa \leq (7-3\sqrt{3})/4$, the subgame-perfect equilibrium locations are

$$y_1^* = \frac{1-\kappa}{3}$$
$$y_2^* = \frac{2+\kappa}{3}$$

and if $(7-3\sqrt{3})/4 < \kappa \leq 1/2$ the subgame-perfect equilibrium locations are

$$y_1^* = \frac{1}{4}\left(2-3\kappa+\sqrt{\kappa^2+4\kappa-2}\right)$$
$$y_2^* = \frac{1}{4}\left(2+3\kappa-\sqrt{\kappa^2+4\kappa-2}\right).$$

Proof subgame-perfect equilibria $1/2 < \kappa \leq 1$

Assume $1/2 < \kappa \leq 1$, then $\kappa > 1-\kappa$. Then, even if firms maximally differentiate their products, the firms' radii of attentive consumers will always overlap, i.e., $y_1+\kappa > y_2-\kappa$. The proof proceeds in three steps: First, we show that a pair (y_1, y_2) such that $y_1 > 1-\kappa$ and/or $y_2 < \kappa$ cannot constitute a subgame-perfect equilibrium. Second, we show that firm 1 never chooses a location $y_1 < (y_2-\kappa)/2$ and firm 2 never chooses a location $y_2 > (1+y_1+\kappa)/2$. Third, we specify the best replies and the subgame-perfect equilibria.

Step 1: Suppose $y_1 \geq 1-\kappa$ and $y_2 > \kappa$:

$$\Pi_1(y_1, y_2) = \frac{1}{18}(y_2-y_1)(3y_2-y_1-4\kappa-2)^2 + \begin{cases} (v-\kappa^2)(y_2-y_1) & \text{if } y_1 \geq \kappa \\ (v-y_1^2)(y_2-\kappa) & \text{if } y_1 < \kappa \end{cases}$$

$$\frac{\partial \Pi_1(y_1, y_2)}{\partial y_1} = \underbrace{\frac{1}{18}\underbrace{(3y_2-y_1-4\kappa-2)}_{<0 \text{ as } 4\kappa+2>3y_2}\underbrace{(-5y_2+3y_1+4\kappa+2)}_{>0 \text{ as } 3y_1+4\kappa+2>5y_2} + \begin{cases} \underbrace{-(v-\kappa^2)}_{<0} & \text{if } y_1 \geq \kappa \\ \underbrace{-2y_1(y_2-\kappa)}_{<0} & \text{if } y_1 < \kappa \end{cases}}_{<0}$$

Suppose $y_1 \geq 1-\kappa$ and $y_2 \leq \kappa$. Then,

$$\begin{aligned}
\Pi_1(y_1,y_2) &= \frac{1}{18}(y_2-y_1)(2+y_1+y_2)^2 \\
\frac{\partial \Pi_1(y_1,y_2)}{\partial y_1} &= \frac{1}{18}\underbrace{(2+y_1+y_2)}_{>0}\underbrace{(y_2-3y_1-2)}_{<0} < 0.
\end{aligned}$$

Therefore, firm 1 never chooses a location $y_1 > 1-\kappa$. Those locations are strictly dominated by $y_1 = 1-\kappa$. By symmetry, firm 2 never chooses a location $y_2 < \kappa$. Those locations are strictly dominated by $y_2 = \kappa$.

Step 2: Thus $y_1 \leq 1-\kappa$ and $y_2 \geq \kappa$. Consequently, the profits of firm 1 and firm 2 are

$$\Pi_1(y_1,y_2) = \frac{1}{2}(y_2-y_1)(y_2-y_1-2\kappa)^2 + (y_2-\kappa)\begin{cases}(v-(y_2-y_1-\kappa)^2) & \text{if } y_1 < \frac{y_2-\kappa}{2} \\ (v-y_1^2) & \text{if } y_1 \geq \frac{y_2-\kappa}{2}\end{cases}$$

$$\Pi_2(y_1,y_2) = \frac{1}{2}(y_2-y_1)(y_2-y_1-2\kappa)^2 + (1-y_1-\kappa)\begin{cases}(v-(y_2-y_1-\kappa)^2) & \text{if } y_2 > \frac{1+y_1+\kappa}{2} \\ (v-(1-y_2)^2) & \text{if } y_2 \leq \frac{1+y_1+\kappa}{2}\end{cases}$$

The profit of firm 1 is strictly increasing for $y_1 < (y_2-\kappa)/2$ and the profit of firm 2 is strictly decreasing for $y_2 > (1+y_1+\kappa)/2$:

$$\begin{aligned}
\frac{\partial \Pi_1(y_1,y_2)}{\partial y_1} &= 2\underbrace{(y_2-y_1-\kappa)}_{>0 \text{ as } y_1<\frac{y_2-\kappa}{2}}\underbrace{(y_2-\kappa)}_{\geq 0} + \frac{3}{2}\underbrace{(y_1-y_2+2\kappa)}_{>0}\underbrace{(y_2-y_1-\frac{2}{3}\kappa)}_{>0 \text{ as } y_1<\frac{y_2-\kappa}{2}} > 0 \\
\frac{\partial \Pi_2(y_1,y_2)}{\partial y_2} &= 2\underbrace{(y_1-y_2+\kappa)}_{<0}\underbrace{(1-y_1-\kappa)}_{\geq 0} + \frac{3}{2}\underbrace{(y_2-y_1-2\kappa)}_{<0}\underbrace{(y_2-y_1-\frac{2}{3}\kappa)}_{>0 \text{ as } y_2>\frac{1+y_1+\kappa}{2}} < 0
\end{aligned}$$

Thus firm 1's optimal location has to be in the interval $[(y_2-\kappa)/2, 1-\kappa]$ and firm 2's optimal location has to be in the interval $[\kappa, (1+y_1+\kappa)/2]$.

Step 3: Next, we derive the best replies for firm 1 and firm 2 with $y_1 \in [(y_2-\kappa)/2, 1-\kappa]$

and $y_2 \in [\kappa, (1+y_1+\kappa)/2]$:

$$\begin{aligned}
\frac{\partial \Pi_1(y_1,y_2)}{\partial y_1} =& -2y_1(y_2-\kappa) - \frac{3}{2}(y_2-y_1-2\kappa)\left(y_2-y_1-\frac{2}{3}\kappa\right) = 0 \\
&\Leftrightarrow y_1(y_2) = \frac{1}{3}\left(y_2 - 2\kappa \pm 2\sqrt{-2y_2^2+5y_2\kappa-2\kappa^2}\right) \\
\frac{\partial \Pi_2(y_1,y_2)}{\partial y_2} =& 2(1-y_2)(1-y_1-\kappa) + \frac{3}{2}(y_2-y_1-2\kappa)\left(y_2-y_1-\frac{2}{3}\kappa\right) = 0 \\
&\Leftrightarrow y_2(y_1) = \frac{1}{3}\left(2+y_1+2\kappa \pm 2\sqrt{-2y_1^2+y_1(4-5\kappa)+\kappa(5-2\kappa)-2}\right).
\end{aligned}$$

Checking the second order condition, we find that the potential maxima are:

$$\begin{aligned}
y_1(y_2) &= \frac{1}{3}\left(y_2 - 2\kappa + 2\sqrt{-2y_2^2+5y_2\kappa-2\kappa^2}\right) \\
y_2(y_1) &= \frac{1}{3}\left(2+y_1+2\kappa - 2\sqrt{-2y_1^2+y_1(4-5\kappa)+\kappa(5-2\kappa)-2}\right).
\end{aligned}$$

But to be the best replies, the potential maxima have to lie inside the interval $[(y_2-\kappa)/2, 1-\kappa]$ and $[\kappa, (1+y_1+\kappa)/2]$. For firm 1:

$$\begin{aligned}
y_1(y_2) \le 1-\kappa \Leftrightarrow y_2 \le \frac{1+3\kappa-2\sqrt{3\kappa-2}}{3} \text{ or } \frac{1+3\kappa+2\sqrt{3\kappa-2}}{3} \le y_2 \\
\frac{y_2-\kappa}{2} \le y_1(y_2) \Leftrightarrow \frac{13-\sqrt{48}}{11}\kappa \le y_2 \le \frac{13+\sqrt{48}}{11}\kappa.
\end{aligned}$$

In addition $y_2 \in [\kappa, (1+y_1+\kappa)/2]$. Therefore, the best reply of firm 1 is

- if $\kappa > \frac{3}{4}$: $y_1^*(y_2) = 1-\kappa$

- if $\frac{2}{3} < \kappa \le \frac{3}{4}$:

$$y_1^*(y_2) = \begin{cases} \frac{1}{3}\left(y_2 - 2\kappa + 2\sqrt{-2y_2^2+5y_2\kappa-2\kappa^2}\right) & \text{if } y_2 \le \frac{1+3\kappa-2\sqrt{3\kappa-2}}{3} \\ 1-\kappa & \text{if } y_2 > \frac{1+3\kappa-2\sqrt{3\kappa-2}}{3} \end{cases}$$

- if $\frac{11}{13+\sqrt{48}} < \kappa \le \frac{2}{3}$: $y_1^*(y_2) = \frac{1}{3}\left(y_2 - 2\kappa + 2\sqrt{-2y_2^2+5y_2\kappa-2\kappa^2}\right)$

- if $\frac{1}{2} < \kappa \leq \frac{11}{13+\sqrt{48}}$:

$$y_1^*(y_2) = \begin{cases} \frac{1}{3}\left(y_2 - 2\kappa + 2\sqrt{-2y_2^2 + 5y_2\kappa - 2\kappa^2}\right) & \text{if } y_2 \leq \frac{13+\sqrt{48}}{11}\kappa \\ \frac{y_2-\kappa}{2} & \text{if } y_2 > \frac{13+\sqrt{48}}{11}\kappa. \end{cases}$$

Similarly, the best reply of firm 2 is, then,

- if $\kappa > \frac{3}{4}$: $y_2^*(y_1) = \kappa$

- if $\frac{2}{3} < \kappa \leq \frac{3}{4}$:

$$y_2^*(y_1) = \begin{cases} \kappa & \text{if } y_1 < \frac{2-3\kappa+2\sqrt{3\kappa-2}}{3} \\ \frac{2+y_1+2\kappa-2\sqrt{-2y_1^2+y_1(4-5\kappa)+\kappa(5-2\kappa)-2}}{3} & \text{if } y_1 \geq \frac{2-3\kappa+2\sqrt{3\kappa-2}}{3} \end{cases}$$

- if $\frac{11}{13+\sqrt{48}} < \kappa \leq \frac{2}{3}$: $y_2^*(y_1) = \frac{2+y_1+2\kappa-2\sqrt{-2y_1^2+y_1(4-5\kappa)+\kappa(5-2\kappa)-2}}{3}$

- if $\frac{1}{2} < \kappa \leq \frac{11}{13+\sqrt{48}}$:

$$y_2^*(y_1) = \begin{cases} \frac{1+y_1+\kappa}{2} & \text{if } y_1 < \frac{11-13\kappa-\sqrt{48}\kappa}{11} \\ \frac{2+y_1+2\kappa-2\sqrt{-2y_1^2+y_1(4-5\kappa)+\kappa(5-2\kappa)-2}}{3} & \text{if } y_1 \geq \frac{11-13\kappa-\sqrt{48}\kappa}{11}. \end{cases}$$

The intersections of the best replies gives the subgame-perfect equilibria.

Thus if $1/2 < \kappa \leq 3/4$, the subgame-perfect equilibrium locations are

$$y_1^* = \frac{1}{4}\left(2 - 3\kappa + \sqrt{\kappa^2 + 4\kappa - 2}\right)$$
$$y_2^* = \frac{1}{4}\left(2 + 3\kappa - \sqrt{\kappa^2 + 4\kappa - 2}\right)$$

and if $3/4 < \kappa \leq 1$, the subgame-perfect equilibrium locations are $y_1^* = 1 - \kappa$ and $y_2^* = \kappa$.

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