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Florian Herold† and Christoph Kuzmics‡

Abstract

Individuals engage in an ex-ante symmetric situation, in which in addition to a symmetric equilibrium there are also asymmetric equilibria. Individuals can assume one of a finite set of payoff irrelevant publicly observable labels and can condition their action choice on their own assumed label as well as the label of their opponent. We study evolutionary (and neutrally) stable strategies of such games. While the formal analysis is similar to the analysis of cheap talk games with evolutionary equilibrium selection, we are here mostly interested in the social structure that underlies such equilibria. For the class of 2×2 games with asymmetric pure strategy equilibria (hawk-dove games) we find a key distinction between two subclasses. While the best-response structure is identical for both subclasses, the evolution is quite different for hawk-dove games in which if you play dove you would prefer the opponent to play hawk (we call these anti-coordination games), and hawk dove games in which you always prefer the opponent to choose dove (we call them conflict games). Two social structures of particular interest are a hierarchical structure and an egalitarian structure. Furthermore, complex social structures composed of simpler substructures can emerge and we characterize their evolutionary stability. We discuss when they are evolutionary stable and the consequences of different structures for welfare.

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1 Introduction

Specialization is a hallmark of economic activity. Sometimes to the benefit of all - the baker needs the farmer to grow the crop and the farmer profits from the baker specializing in producing bread, cakes, and pastries. Other specializations - say specializing in theft - may evolve detrimental to other peoples benefit - but still evolve as equilibrium behavior. While farmers would prefer nobody specializing in theft, once there are farmers planting crop, there may be an incentive to specialize in stealing some crop or in other methods of rent extraction. On the other hand, if there are no farmers and nobody from which you could extract rents, these specializations become pointless. We want to investigate the evolution of different roles in such interactions. Ex ante people are symmetric and could specialize in any task or role, but the structure of the game is such that mutual best-responses in pure strategies are asymmetric and there exist several asymmetric pure strategy equilibria.

We investigate under which circumstances people endogenously evolve into assuming different roles in situations when they start in symmetric positions and face a symmetric interaction. The canonical example we consider is an interaction which has the structure of a symmetric 2×2 game in which both pure strategy equilibria are asymmetric. The best-responses to the opponents' pure strategies are as in the hawk-dove game: The best response to one pure strategy is the other pure strategy. We are interested in games where the two players earn different payoffs in the asymmetric equilibria and the two players therefore prefer different pure strategy equilibrium play. Consider the base game

$$\begin{array}{c|cc} & H & D \\ \hline H & c & a \\ D & b & d \end{array},$$

W.l.o.g. we assume $a > b$ (otherwise simply switch the labels H and D). The key restrictions we impose on our game are $b > c$ and $a > d$.

This class of games is the canonical example in the evolutionary game theory literature for a game in which the evolutionary stable strategy in a one population model is very different from the evolutionary stable strategies in two-population models. In the one-population model both players playing the game are drawn from the same population and the unique evolutionary stable strategy is the symmetric mixed Nash equilibrium. In the two population model the mixed Nash equilibrium is not even a neutrally stable strategy (and thus neither evolutionary stable) and the only evolutionary stable strategies are the two asymmetric equilibria. The cause for this drastic difference in results is that in the one population model players cannot make their play contingent on their player-position and thus it is essentially impossible to play an asymmetric mixed strategy profile. When we move from the one population model to the two population model we do however not only allow players to play contingent on their position, but we also assume that evolutionary competition occurs only within the population of each player-positions separately. We want to consider a framework in which players have different labels which do not affect payoffs directly yet allow matched players with different labels to play asymmetrically. Evolutionary competition however is still evaluated across the entire population. Thus, not only actions conditional on labels evolve, but also the distribution of labels itself. More precisely we consider neutrally stable strategies (NSS) and evolutionary stable strategies (ESS) of the meta-game in which players can choose their label and an action that can condition on the opponent's label or type.[1]

An important observation (Lemma 2) for all further results is that in any NSS matched players with different labels must anti-coordinate, while players must play the mixed Nash-equilibrium when matched with their own label.[2] To develop a better intuition for the coming results one can imagine that for the exiting labels a social-structure develops that specifies for each combination of different labels which of the two asymmetric Nash equilibria is the convention. Given such a social structure, labels that do earn lower payoffs become less frequent over time. Hence in a NSS of the meta-game all labels in the supports of the NSS must earn the same payoff given this distribution of labels and given a social structure consistent with Lemma 2.

For a situation with two potential labels only, there is - up to a permutation in labels - only one social structure that is consistent with Lemma 2. A 'top' label will play H against the other 'bottom' label. A key distinction emerges from the evolutionary analysis between two (sub)classes. While all symmetric base-games we consider have the same best-response structure, it is interesting to note that these two classes of games are economically quite different games. We call the first sub-class the class of conflict games. In a game of conflict you always prefer the opponent to play the dove strategy D, independently of your own choice of action.[3] We call the second sub-class anti-coordination games. In anti-coordination games a player who did commit to playing action D would actually prefer the opponent to play action H. Thus, this class of games corresponds rather to a game of specialization in which, for instance, the two players act in a team and success

[1]Note that types play a somewhat different role here compared to the literature on the evolution of preferences under observability in which a type implies some subjective preferences and therefore certain strategic behavior (compare e.g. Dekel, Ely, and Yilankaya (2007) and Herold and Kuzmics (2009)). Here choosing a type does not constrain the strategies you may choose.

[2]The result that players with different roles must anti-coordinate is already present in Selten (1980), but he keeps these roles exogenous.

[3]The interaction in a conflict game corresponds e.g. to the Hawk-Dove game described in Maynard Smith (1982) in which the reward is equally shared if both animals retreat.

requires two different skills. One skill is more attractive to acquire than the other, but given you acquired one particular skill, you prefer that your partner has acquired the complementary skill necessary for joint success. We name such interactions "anti-coordination games". It turns out that in conflict games only the top label can be in the support of an NSS and despite the existence of the two labels we are essentially back to the result of the one-population model without labels. For anti-coordination games, in contrast, both labels are present in the support of the unique NSS, but bottom types in a lower proportion than top types. Thus the inefficiency is reduced, but some inefficiency remains in the NSS.

For more then two labels Lemma 2 is consistent with several social structures: For instance one might have a structure in which labels have a transitive order and higher labels play H against lower labels. We call this a hierarchical structure. Yet, also some circular structures are possible, which for an odd number of labels can lead to egalitarian outcomes (and for an even number of labels to outcomes approximately egalitarian). For a specific anti-coordination game Hurkens and Schlag (2002) showed this already and they conjecture that their result extends to other anti-coordination games. We briefly confirm this conjecture, after we provide general characterizations of ESS and NSS in our general setup.

Then we focus our analysis on games of conflict. First, in games of conflict no hierarchical structure can form an NSS with more than one label in its support. Second another interesting distinction emerges. In conflict games in which the payoff d (if both players play dove) is below the average payoff the two players obtain in the asymmetric equilibrium, an egalitarian types structure forms an ESS.

For a large number of labels complex social structures composed of stable substructures can be evolutionary stable and we provide a characterization of when such group substructures are stable. Finally, we summarize some key results contingent on the parameter d and discuss the welfare implications. It turns out that in our setting an egalitarian social structure is good for a society - even from an efficiency point of view. Intuitively, an egalitarian social structure makes all roles in society equally attractive and thus helps to avoid situations where too many players choose the more attractive roles in society.[4]

1.1 Related Literature

The hawk-dove games was one of the first games analyzed in in evolutionary game theory starting with the seminal work by Maynard Smith and Price (1973) and Maynard Smith (1982). Selten (1980) already discusses explicitly

[4]Results in this spirit have been established for repeated symmetric games by Bhaskar (2000) and Kuzmics, Palfrey, and Rogers (2014), where the promise of an egalitarian continuation play induces efficient randomization in early rounds of play.

that if there is some asymmetry between the opponents or if they have different roles, only the asymmetric pure strategy equilibria are evolutionary stable. While in Selten (1980) these different roles are given exogenously, we are interested here in their endogenous evolution.

The most closely related literature are evolutionary papers on cheap talk games. We can redefine payoff irrelevant labels as cheap talk messages and search for the NSS or ESS of these games. An early paper that discusses an anti-coordination game with a specific type of cheap talk messages is Farrell (1987). He allows only for a specific type of communication corresponding to our hierarchical structure in anti-coordination games and analyzes the corresponding Nash equilibria. Most of this related cheap talk literature, such as Robson (1990), Wärneryd (1993), Sobel (1993), Blume, Kim, and Sobel (1993), Schlag (1993), Schlag (1995), Kim and Sobel (1995), Bhaskar (1998), Banerjee and Weibull (2000), Hurkens and Schlag (2002), focus on coordination games and in how far cheap talk will - or will not help to select against in-efficient equilibria. Most closely related formal setup to our work is Hurkens and Schlag (2002) and Banerjee and Weibull (2000). While both papers focus on coordination games, the work by Hurkens and Schlag (2002) has also a section on a task allocation game which falls in our subclass of anti-coordination games. For this task-allocation game they find necessary conditions for ESS corresponding to our conditions (a),(b) and(d) in Lemma 3. Our lemma adds to their necessary condition by providing a full characterization of ESS. In their proofs of the lowest and highest payoffs in ESS they also use constructions corresponding to what we call hierarchical type structure and respectively egalitarian or approximate egalitarian type structure. They also conjecture that that these results extend to a larger class of anti-coordination games. We briefly conform this conjecture. More importantly we analyze all 2×2 games with the best response structure of hawk-dove games and find the key distinction between anti-coordination games and conflict games not discussed in the literature before. Thus, from the perspective of the cheap talk literature, we complete the analysis of the class of all 2×2 games. We think another perspective is perhaps even more important: We endogenize the evolution of roles. While Selten's argument that different roles lead to asymmetric pure strategy equilibria is confirmed, it is a subtle question how likely different labels meet in equilibrium for a large number of potential labels. For hierarchical type structures the top label often dominates other types or is played more often than would required by efficiency. Only for more subtle circular type structures opponents are likely to have different labels in the evolving equilibrium. The emerging stable social structures (or type structure) are interesting in their own right. One interesting conclusion is that an egalitarian social structure can enhance efficiency relative to a hierarchical social structure, because in an hierarchical structure too many players want to have the top label which prevents anti-coordination.

We come back to the relation to Farrell (1987), Banerjee and Weibull (2000), and Hurkens and Schlag (2002) and discuss it in more detail in Section 4 after we derived our results.

2 Model

This paper studies a special class of symmetric two-player two-strategy games with a pre-game cheap-talk phase. We call the two by two game the **base game** and the base game plus the cheap-talk phase the **meta game** as in Banerjee and Weibull (2000).

2.1 The Base Game

The **base game** is a symmetric 2x2 game given by the payoff matrix

$$\begin{array}{c|cc} & H & D \\ \hline H & c & a \\ D & b & d \end{array},$$

with the following restrictions.[5] W.l.o.g. we assume that all payoffs are non-negative, i.e. $a, b, c, d \geq 0$ (one could always add a constant to all payoffs without affecting the incentives in the game). W.l.o.g. we assume $a > b$ (if not we would simply switch labels H and D).

The key restrictions we impose on our game are $b > c$ and $a > d$. These last two restrictions imply that the best response to H is D and to D is H. This means we rule out dominant strategy games and coordination games and this is all we rule out.[6] We shall call the class of games as described above **(general 2×2) hawk-dove games**.

The results in this paper will differ crucially for two (disjoint and jointly exhaustive) subclasses of the class of hawk-dove games. The crucial distinction is how b compares to d. When $b \leq d$ a player always prefers the opponent to play the dove strategy D, independently of her own choice of action. We shall call such games **conflict games**. In contrast, when $b > d$ then a player who did commit to playing action D would actually prefer the opponent to play action H. We call such games **anti-coordination games**.

[5]Throughout the paper we ignore the possibilities of payoff-ties. Generically there are no payoff-ties and this simplifies the exposition without affecting the main message.

[6]Symmetric 2x2 games are typically classified by the best responses into four categories: two classes of dominant strategy games (efficient dominant strategy games and prisoners dilemma games), coordination games, and hawk-dove (also chicken) games (Compare e.g. Weibull (1995) or Eshel, Samuelson, and Shaked (1998). Dominant strategy games are of no interest for our purpose. In such games in our model evolution will always lead to everyone playing the dominant action. Players may send messages but they will not impact play. Coordination games are of interest in our context, but have already been subjected to a thorough analysis in Banerjee and Weibull (2000) and Hurkens and Schlag (2002), among others.

The following lemma collects a few immediate and mostly well-known facts about this class of games, which are later useful for the further analysis of the meta game.

Lemma 1. *A hawk-dove game (with parameters a, b, c, d satisfying $a > b > c$ and $a > d$) has the following properties.*

1. *There are exactly two pure strategy Nash equilibria. These are asymmetric. One player plays H and the other D.*

2. *The game has a unique symmetric equilibrium which is in mixed strategies with probability x^* placed on H, where $x^* = \frac{a-d}{a-d+b-c}$.*

3. *The expected payoff (to both players) in the symmetric (mixed strategy) equilibrium is given by $u^* = \frac{ab-cd}{a-d+b-c}$.*

4. *The payoff in the symmetric equilibrium, u^*, is lower than b, the low payoff in the asymmetric equilibria, if and only if $d < b$ (i.e. if and only if the game is an anti-coordination game).*

5. *There exists a strategy limiting the opponent's expected payoff to $\min\{u^*, b\}$. In anti-coordination games this is achieved by playing x^*, in conflict games by playing $x = 1$ (hawk).*

6. *Keeping the parameters a, b, c fixed the mixed equilibrium payoff u^* is strictly increasing in the parameter d, with $\lim_{d \to -\infty} u^* = c$ and $\lim_{d \to a} u^* = a$.*

7. *Keeping a, b, c fixed, there is a unique cutoff value $\bar{d} \in (\frac{a+b}{2}, a)$ for which $u^*(\bar{d}) = \frac{a+b}{2}$, specifically $\bar{d} = \frac{a^2+b^2-c(a+b)}{a+b-2c}$.*
   The payoff in the symmetric equilibrium u^* is higher than the average of the two payoffs in an asymmetric equilibrium $\frac{1}{2}(a+b)$ if and only if $d > \bar{d}$.

Points 1-3 of this Lemma are commonly known and their proofs omitted. The remaining points are not usually emphasized. Their straightforward proofs are given in Appendix A.1. In particular Point 4 will turn out important for the further evolutionary analysis of the meta game.

2.2 The Meta Game

Let $G = (A, u)$ be any two player hawk-dove game (with $a > b > c$ and $a > d$), the *base game*. Before players play the base game they can freely, i.e. without cost, adopt one of finitely many (commonly observable) roles or types (or can send one of finitely many commonly distinguishable messages). The finite set of types (roles, messages) is given by type space Θ.

Types are, therefore, payoff-irrelevant, but perfectly observable and players can condition their play on the opponents type $\theta' \in \Theta$. The formal setup is, thus, almost identical with that of Banerjee and Weibull (2000) and Hurkens and Schlag (2002), except that we study the entire class of hawk-dove base-games and focus on the resulting social-structures, while their focus is to study equilibrium selection in coordination games and in an anti-coordination game.[7]

Let $F = \{f : \Theta \to A\}$ the (finite) set of action-functions. Then $f(\theta')$ provides the action that a player chooses against an opponent of type θ'.

Define $S = \Theta \times F$ as the (finite) set of pure strategies of the meta game. Correspondingly, let $\Delta(S)$ be the set of mixed strategies of the meta game and u the properly expanded payoff function. Thus $\Gamma = (S, u)$ defines the finite meta-game.

A mixed strategy $\sigma \in \Delta(S)$ thus induces both a probability distribution over adopted types as well as, for each adopted type, a probability distribution over actions. For the purpose of stating (and proving) our results it is useful to have formal expressions of these distributions.

We define $\sigma(\theta) \equiv \sum_{f \in F} \sigma(\theta, f)$, the marginal probability of a player, using mixed strategy $\sigma \in \Delta(S)$, adopting type $\theta \in \Theta$. Furthermore we denote the (conditional) probability that a player of type θ, given strategy $\sigma \in \Delta(S)$ plays H against an opponent of type θ' by $x_\theta(\theta') = \frac{\sum_{f \in F, f(\theta')=H} \sigma(\theta,f)}{\sigma(\theta)}$.[8]

Note that any $\sigma \in \Delta(S)$ uniquely determines $\sigma(\theta)$ for every $\theta \in \Theta$ and $x_\theta(\theta')$ for all $\theta, \theta' \in \Theta$. The converse is not generally true.[9] However, in order to compute the expected payoff $u(\sigma, \tilde{\sigma})$ which a player with strategy σ obtains against an opponent with strategy $\tilde{\sigma}$ it is sufficient to know $\sigma(\theta)$

[7] There is one formal, but non-substantive, difference between the way we define pure strategies in the meta-game and the way this is done in Banerjee and Weibull (2000). They allow players to condition on both their opponent's as well as their own type. We prefer to reduce the number of strategies, without losing anything, by allowing players to condition only on their opponent's type. We thus, follow Schlag (1993), Schlag (1995) and Hurkens and Schlag (2002), in this respect. For a discussion of this issue see pages 11-12 in Banerjee and Weibull (2000). One advantage of using this reduced form approach is that it makes more clear when a failure of evolutionary but not neutral stability is simply due to a large number of equivalent strategies or due to a more fundamental problem intrinsic to the game under analysis.

[8] We should perhaps indicate the dependence of $x_\theta(\theta')$ on σ by writing $x_\theta^\sigma(\theta')$. The context should be sufficient for clarity. We shall, for instance, have σ and σ' and then correspondingly $x_\theta(\theta')$ and $x'_\theta(\theta')$.

[9] Consider for instance a meta game with $\Theta = \{T, B\}$ and the corresponding set of action functions $F = \{f_{HH}, f_{HD}, f_{DH}, f_{DD}\}$, where f_{a_T,a_B} is the action function with $f(T) = a_T$ and $f(B) = a_B$ for $a_T, a_B \in \{H, D\}$. Then the two strategies $\sigma = \frac{1}{2}(T, f_{HH}) + \frac{1}{2}(T, f_{DD})$ and $\tilde{\sigma} = \frac{1}{2}(T, f_{HD}) + \frac{1}{2}(T, f_{DH})$ which are different from each other but lead to the same $\sigma(\theta) = \tilde{\sigma}(\theta)$ for every $\theta \in \Theta$ and $x_\theta(\theta') = \tilde{x}_\theta(\theta')$ for all $\theta, \theta' \in \Theta$.

and $\tilde{\sigma}(\theta)$ for every $\theta \in \Theta$ and $x_\theta(\theta')$ and $\tilde{x}_\theta(\theta')$ for all $\theta, \theta' \in \Theta$:

$$\begin{aligned} u(\sigma, \tilde{\sigma}) = \textstyle\sum_{\theta, \theta' \in \Theta} \sigma(\theta)\tilde{\sigma}(\theta') \quad & \big[x_\theta\left(\theta'\right)\tilde{x}_{\theta'}\left(\theta\right)c \\ & +x_\theta\left(\theta'\right)\left(1-\tilde{x}_{\theta'}\left(\theta\right)\right)a \\ & +\left(1-x_\theta\left(\theta'\right)\right)\tilde{x}_{\theta'}\left(\theta\right)b \\ & +\left(1-x_\theta\left(\theta'\right)\right)\left(1-\tilde{x}_{\theta'}\left(\theta\right)\right)d\big] \end{aligned}$$

We thus call two strategies $\sigma \in \Delta(S)$ and $\hat{\sigma} \in \Delta(S)$ **equivalent** if $\sigma(\theta) = \hat{\sigma}(\theta)$ for all $\theta \in \Theta$ and $x_\theta(\theta') = \hat{x}_\theta(\theta')$ for all $\theta, \theta' \in \Theta$. We can now define the corresponding equivalent classes. It will however turn out that all strategies that satisfy some necessary conditions for neutral stability or evolutionary stability are unique in their equivalent class and we do not further need to worry about this issue for our results.

2.3 The Solution Concept

We can now use standard concepts such as Evolutionary Stable Strategy (ESS) and Neutrally Stable Strategy (NSS) from evolutionary game theory and apply them to our meta game.[10] One way to define these concepts is as follows.

Definition 1. *A strategy of the meta game $\sigma \in \Delta(S)$ is a* **neutrally stable strategy (NSS)** *if and only if the following two conditions hold:*

$$\begin{array}{lll} (1) & u(\sigma, \sigma) \geq u(\sigma', \sigma) & \forall \sigma' \in \Delta(S) \\ (2) & u(\sigma, \sigma) = u(\sigma', \sigma) \;\; \Rightarrow u(\sigma, \sigma') \geq u(\sigma', \sigma') & \forall \sigma' \neq \sigma. \end{array}$$

Strategy $\sigma \in \Delta(S)$ is an **evolutionary stable strategy (ESS)** *if and only if the same two conditions hold and the last inequality is strict.*

We shall refer to condition (1) as the first order condition or FOC and condition (2) as the second order condition or SOC. Note that any ESS is also an NSS.

3 Results

3.1 Preliminaries

Lemma 2. *For any neutrally stable strategy (NSS) $\sigma \in \Delta(S)$ of the meta game of any hawk-dove base game the following necessary conditions must hold:*

(a) For all $\theta \in \Theta$ with $\sigma(\theta) > 0$: $x_\theta(\theta) = x^*$.

[10] See e.g. Chapter 2 of Weibull (1995) for a textbook treatment of these definitions and concepts.

(b) For all $\theta, \theta' \in \Theta$ with $\sigma(\theta) > 0$ and $\sigma(\theta') > 0$:
$x_\theta(\theta') = 1 - x_{\theta'}(\theta) \in \{0, 1\}$.

In other words in any neutrally stable strategy (and therefore also in any evolutionary stable strategy) of the meta game every two different types which are chosen with positive probability must anti-coordinate on $\{H, D\}$ or $\{D, H\}$ when matched against each other. When matched with their own type the mixed symmetric equilibrium of the base game must be played.

The detailed proof of Lemma 2 is given in Appendix A.2. The result, however, is in some sense well known. We know from Maynard Smith (1982), see e.g. (Weibull 1995, pp.40-41) for a textbook treatment that in the single population case (i.e. here, whenever two individuals of the same type meet) the only evolutionary stable outcome is the symmetric mixed equilibrium. We known from Selten (1980) that in the multiple population model (i.e. here, whenever two individuals of different types meet) the only evolutionary stable outcome must be a strict, and, hence, pure and possibly asymmetric equilibrium. The proof of Lemma 2 follows essentially the arguments of the two mentioned results given the language of our model.

Note that Lemma 2 is silent about the distribution over types in Θ in an NSS. Understanding this is where the contribution of this paper lies and this is what we investigate in the next three (sub)sections.

These necessary conditions for NSS must also be necessary conditions for ESS, of course. For a specific anti-coordination game (task allocation game) Hurkens and Schlag (2002) provided corresponding necessary conditions for evolutionary stability and showed that in any ESS all labels must be played with positive probability (refer to their Lemma 2(ii)). Adding an additional condition (which holds for all anti-coordination games and some but not all conflict games) we can provide a nice characterization of ESS:

Lemma 3. *Let $|\Theta| \geq 2$. A strategy $\sigma \in \Delta(S)$ of the meta game of any hawk-dove base game is an ESS if and only if the following five conditions are all satisfied.*

(a) For all $\theta \in \Theta$: $x_\theta(\theta) = x^*$.

(b) For all $\theta, \theta' \in \Theta$:
$x_\theta(\theta') = 1 - x_{\theta'}(\theta) \in \{0, 1\}$.

(c) For all $\theta \in \Theta$: $\sigma(\theta) > 0$ (all labels are played with positive probability).

(d) All strategies in the support of σ earn the same payoff: $u(s, \sigma) = u(\sigma, \sigma)\ \forall s \in Supp(\sigma)$.

(e) $\frac{a+b}{2} > d$.

In contrast to ESS, neutrally stable strategies do not necessarily play every label with positive probability. Yet, if all labels are played with positive probability, then the next lemma provides a very similar characterization for all such NSS with full label-support:

Lemma 4. *Let $|\Theta| \geq 2$ and $\sigma(\theta) > 0$, for all $\theta \in \Theta$ (all labels are played with positive probability). Then the following conditions are necessary and jointly sufficient to establish that the strategy $\sigma \in \Delta(S)$ of the meta game of any hawk-dove base game is a neutrally stable strategy (NSS):*

(a) For all $\theta \in \Theta$: $x_\theta(\theta) = x^*$.

(b) For all $\theta, \theta' \in \Theta$: $x_\theta(\theta') = 1 - x_{\theta'}(\theta) \in \{0, 1\}$.

(c) All strategies in the support of σ earn the same payoff: $u(s, \sigma) = u(\sigma, \sigma)$ $\forall s \in Supp(\sigma)$.

(d) $\frac{a+b}{2} \geq d$.

Note that Lemma 4 is mute about strategies σ' for which there is a $\theta \in \Theta$ such that $\sigma'(\theta) = 0$: these may - or may not - be an NSS. The next lemma gives a necessary condition for NSS in case we do not have full label-support.

Lemma 5. *Let σ be a strategy of the meta game of any hawk-dove base game. For such σ let Θ_S denote the set of labels $\theta \in \Theta$ with $\sigma(\theta) > 0$ and let Θ_M denote the set of labels $\theta' \in \Theta$ with $\sigma(\theta') = 0$. Let furthermore $\sigma|\Theta_S$ denote the strategy σ restricted to the set of labels Θ_S.*

A necessary condition for σ to be an NSS of the meta game with set of labels Θ is that $\sigma|\Theta_S$ is an NSS of the meta game with the same hawk-dove base game with the set of labels Θ_S.

We can also provide a sufficient condition for NSS without full label support, by extending any full label support equilibrium from a smaller set of labels Θ_S, with $2 \leq |\Theta_S| < |\Theta|$, to the entire set of labels Θ.

Lemma 6. *Let $\sigma|\Theta_S$ be a NSS of the meta game of a hawk-dove base game with full label support on the set of labels Θ_S, with $|\Theta_S| \geq 2$. Then for any $\Theta \supset \Theta_S$ there exists a strategy σ that is an NSS of the meta game with the same hawk-dove base game with $\sigma(\theta) = 0$ for all $\theta \notin \Theta_S$, $\sigma(\theta) = \sigma|\Theta_S(\theta)$ for all $\theta \in \Theta_S$ and identical $x_\theta(\theta')$ for all $\theta, \theta' \in \Theta_S$.*

In contrast to the cheap talk literature our focus is more on the social structure, or type structure, that develops in the meta game. It is therefore useful to imagine a two-speed dynamics: In the short run the distribution of labels is given. A fast learning dynamics leads to behavior consistent with Conditions (a) and (b) of Lemma 2: Players play the mixed base-game equilibrium against their own label and, when two different labels meet, there is a clear convention who plays H and who plays D. This is what we call a pre-stable type structure.

Definition 2. *Consider a meta game with a hawk-dove base game and a set of labels Θ. Given a meta-game strategy $\sigma \in \Delta(S)$, we call the induced type behavior x with $x_\theta(\theta') \in \Delta(A)$ the behavior of type θ when meeting type θ' as defined in Section 2.2, the induced* type structure.

- *A* **pre-stable type structure** *is a type structure satisfying the following conditions:*

  (a) For all $\theta \in \Theta$: $x_\theta(\theta) = x^*$.

  (b) For all $\theta, \theta' \in \Theta$: $x_\theta(\theta') = 1 - x_{\theta'}(\theta) \in \{0, 1\}$.

- *The induced* **type game** *of a pre-stable type structure is a 2-player normal-form game with $|\Theta| \times |\Theta|$ payoff-matrix T defined by $T_{\theta\theta} \equiv u^*$ for all $\theta \in |\Theta|$ and for all $\theta' \neq \theta$: $T_{\theta\theta'} = a$ if $x_\theta(\theta') = 1$ and $T_{\theta\theta'} = b$ if $x_\theta(\theta') = 0$.*

For any given pre-stable type structure we can now investigate how the composition of labels evolves (imagine at a slow speed) in the corresponding reduced form "type game". First, we investigate which distribution of labels leads to an Nash-equilibrium in this type game. Then it is straightforward to check whether the corresponding strategies are stable in the meta game. The relationship is summarized in the following lemma:

Lemma 7. *Consider the meta game of any hawk-dove base game with finite set of types Θ, with $|\Theta| \geq 2$.*

(a) There exists an evolutionary stable strategy (ESS) $\sigma \in \Delta(S)$ of the meta game with a certain type structure, if and only if the type structure is pre-stable, the corresponding type game has a full support Nash-equilbrium, and $\frac{a+b}{2} > d$.[11]

(b) There exists a neutrally stable strategy (NSS) $\sigma \in \Delta(S)$ of the meta game with a certain type structure and $\sigma(\theta) > 0$ for all $\theta \in \Theta$, if and only if the type structure is pre-stable, the corresponding type game has a full support Nash-equilbrium, and $\frac{a+b}{2} \geq d$.

The proof follows immediately from Lemmata 3 and 4.

Note furthermore, that if σ is an ESS of the meta game, then it must be the unique ESS with this type structure. To see this note that if σ is an ESS

[11] Within the type game the ESS condition is only $\frac{a+b}{2} > u^*$. Yet, for stability in the meta game, we need the more restrictive condition $\frac{a+b}{2} > d$: there are hawk dove base-games with $\frac{a+b}{2} < d < \bar{d}$ for which egalitarian structures (defined later) form no NSS, for example $a = 3, b = 1, c = 0$, and $d = 2.2$. Then $\frac{a+b}{2} = 2$, $\bar{d} = 2,5$, $x^* = \frac{4}{9}$, and $\sigma = \frac{1}{27}(4,4,4,5,5,5)$ (where we restrict σ to the mixtures of pure best responses, and write first the 3 optimal pure strategies playing H against its own label, and then the three optimal pure strategies playing D against its own label). Then, e.g., the mutant strategy $\mu = \frac{1}{27}(4,4,4,8,2,5)$ violates the SOC of NSS (and thus also for ESS).

of the meta game, then the corresponding strategy must also be an ESS of the corresponding type game. In the type game it is a full support ESS and must therefore be unique.[12] But then no other strategy of the meta game with the same type game can form an ESS.

To check whether there exist NSS of the meta game a certain pre-stable type structure without full label support, it is still useful to look at Nash equilibria the type game (without full support), yet we need to check that all best responses in the meta game, do perform weakly worse against themselves than the NSS strategy against this mutant strategy.

3.2 The meta game with two types

This section considers the special case when only two types (or roles) are available. This case is relatively simple to analyze and, yet, already demonstrates a key insight of this paper, which is then generalized in Section 3.3.

Proposition 1. *Let $\Theta = \{T, B\}$ and consider the meta game Γ with base game given by a hawk-dove game (with $a > b > c$ and $a > d$).*

(a) If $b \leq d$, i.e. the base game is a conflict game, then $\sigma \in \Delta(S)$ is a neutrally stable strategy if and only if there is a $\theta \in \Theta$ with the property that $\sigma(\theta) = 1$, $x_\theta(\theta) = x^*$, and $x_\theta(\theta') > x^*$ for $\theta' \neq \theta$. Furthermore, there exists no evolutionary stable strategy.

(b) If $b > d$, i.e. the base game is an anti-coordination game, then for $\sigma \in \Delta(S)$ to be a neutrally stable strategy it must satisfy $\sigma(\theta) > 0$ for both $\theta \in \{T, B\}$. In fact it is an NSS if and - up to a permutation of labels - only if $\sigma(T) = \sigma_T^*$, $x_T(T) = x^* = x_B(B)$, $x_T(B) = 1$, and $x_B(T) = 0$, where $\sigma_T^* \equiv \frac{(a-c)(a-d)}{a^2+2cd-ad-ac-bd-bc+b^2}$. These neutrally stable strategies are also evolutionary stable strategies (ESS).

The proof of Proposition 1 is given in Appendix A.8. A simple sketch of the proof is as follows. By Lemma 2 we know that in an NSS when two people of the same type meet they must use the symmetric equilibrium x^* and when two people of different types meet they must play one of the the two asymmetric equilibria.[13] All that remains then is to identify the NSS equilibrium probabilities of adopting types. In essence, we have the following, and subject to relabelling T and B unique, "type game".

[12] See e.g. (Weibull 1995), page 41. Any interior ESS must be unique: Since it has full support, all strategies are best responses against the ESS strategy. Hence, the SOC always applies and the ESS strategy must earn more against any other mutant strategy then the mutant strategy against itself. Hence, none of these other strategies can form an ESS.

[13] In this sketch of the proof we ignore that Lemma 2 implies this behavior only for positive probability types. The proper proof in Appendix A.8 is more careful.

	T	B
T	u^*	a
B	b	u^*

This game has a dominant "strategy" T if and only if u^*, the payoff in the mixed strategy equilibrium of the underlying base game, exceeds b, and u^* exceeds b if and only if the underlying base game is an conflict game by Lemma 1.4. This implies that if the underlying base game is an conflict game any neutrally stable strategy must put probability weight on a single type, here T. Thus, in conflict games we do not expect two subpopulations of types to emerge and we do expect the symmetric (and mixed strategy) equilibrium of the base game to be played in all encounters.

On the other hand, if the base game is an anti-coordination game, again by Lemma 1.4, $u^* < b$, and the "type game" is again in the class of hawk-dove games, in fact, it is another anti-coordination game. Here, we do expect, in any NSS of the meta game, that two subpopulations evolve.

3.3 The meta game with any finite number of types

In this section we consider an arbitrary finite type space Θ.

The following definitions prove useful for our further analysis.

Definition 3. *Consider a meta-game strategy $\sigma \in \Delta(S)$ and an induced pre-stable type structure with $x_\theta(\theta) = x^*$ for all $\theta \in \Theta$.*

- *If there is an order of types $\succ$ such that $x_\theta(\theta') = 1$ (plays H) if $\theta \succ \theta'$ and $x_\theta(\theta') = 0$ (plays D) if $\theta' \succ \theta$, then x is called a* ***hierarchical*** **type structure**.
- *Suppose $|\Theta|$ is odd. If $x_\theta(\theta') = 1$ (plays H) for exactly half of all types $\theta' \neq \theta$ and $x_\theta(\theta') = 0$ (plays D) for the other half of all types $\theta' \neq \theta$, then x is called an* ***egalitarian*** **type structure**.[14]
- *Suppose $|\Theta| \geq 4$ is even, i.e. there is a natural number $k > 1$ such that $|\Theta| = 2k$. If for exactly k types $x_\theta(\theta') = 1$ (plays H) for exactly half of all types $\theta' \neq \theta$ and $x_\theta(\theta') = 0$ (plays D) for the other $k-1$ of all types $\theta' \neq \theta$, and if for the remaining k types $x_\theta(\theta') = 1$ (plays H) for exactly $k-1$ of all types $\theta' \neq \theta$ and $x_\theta(\theta') = 0$ (plays D) for the other k of all types $\theta' \neq \theta$ (and if the resulting type game has a full support Nash-equilibrium), then x is called a* ***approximate egalitarian*** **type structure**.[15]

[14]If $|\Theta|$ is even, then an exactly egalitarian type structure is obviously impossible. Yet, one can define an close to egalitarian structure in which half the types play exactly one more time H than D and the other half one more time D then H. The results would be very similar to the results we obtain for odd numbers of types, but would complicate the arguments and notation.

[15]The assumption of a full support Nash-equilibrium of the type game is probably already implied, but we still need to prove that.

Note that these definitions are not empty, meaning that we can indeed construct a strategy $\sigma \in \Delta(S)$ with a hierarchical type structure and we can also construct one with an egalitarian type-structure, provided the number of types in Θ is odd.[16] If the number of types in Θ is even we can also construct a strategy $\sigma \in \Delta(S)$ with a close to egalitarian type structure.[17]

As an example consider the case $\Theta = \{T, M, B\}$, i.e. $|\Theta| = 3$. The hierarchical and egalitarian, respectively, type structures can be represented in terms of the induced "type game" given by the following two matrices.

$$\begin{array}{c|ccc}
 & \text{T} & \text{M} & \text{B} \\
\hline
\text{T} & u^* & a & a \\
\text{M} & b & u^* & a \\
\text{B} & b & b & u^*
\end{array}
\qquad
\begin{array}{c|ccc}
 & \text{T} & \text{M} & \text{B} \\
\hline
\text{T} & u^* & a & b \\
\text{M} & b & u^* & a \\
\text{B} & a & b & u^*
\end{array}$$

hierarchical egalitarian

For the hierarchical type structure type T is the "top type" and plays hawk (H) against all other types. Type M is the middle type, who plays dove (D) against type T and hawk (H) against type B. Type B is the bottom type, who plays dove (D) against all other types.

In the egalitarian type structure all types play hawk (H) against one other type and dove (D) against the remaining other type. Thus, they are all in equal or "egalitarian" positions.

The next two lemmata investigate the stability properties of hierarchical and egalitarian type structures in our two classes of games, games of anti-coordination and conflict games.

Proposition 2. *Let* $|\Theta| \geq 2$.

(a) There exists an ESS of the meta game with an hierarchical type structure if and only if the base game is an anti-coordination game, i.e. $b > d$. *This ESS is the unique symmetric equilibrium with hierarchical type structure.*

(b) There exists an NSS of the meta game with an hierarchical type structure for all hawk-dove base-games. For anti-coordination games this NSS is the unique ESS with hierarchical type structure with full label

[16]There exist several ways to visualize an egalitarian type structure. For instance, one could arrange types in Θ on a circle such that each type θ plays hawk (H) against the $(n-1)/2$ types located clockwise from θ and plays dove (D) against all other types.

[17]One construction can again be visualized by arranging Types in Θ on a circle such that the first k of the $2k$ types θ plays hawk (H) against the k types located clockwise from θ and plays dove (D) against the other types. Each type θ' from the remaining $k+1$ to $2k$ types plays hawk (H) against the $k-1$ types located clockwise from θ' and dove (D) against the others. It only remains to check that this type structure has a full support equilibrium. In one of their proofs Hurkens and Schlag (2002) use already a similar construction to the one used in this footnote.

support from above. For conflict games this hierarchical NSS has only strategies in its the support that play exclusively the top-label.

For conflict games the unique symmetric Nash-equilibrium of the hierarchical type game puts all weight on the top-label strategy. The corresponding strategy cannot be an ESS of the meta game but is still a NSS.

Proposition 3. *Let $n \equiv |\Theta| \geq 3$ be an odd number. There exists a strategy of the meta-game of any hawk-dove base-game that induces an egalitarian type structure and has full label support. In this egalitarian equilibrium each strategy receives an average payoff of*

$$v_n \equiv \frac{u^*}{n} + \frac{n-1}{n}\frac{a+b}{2}. \tag{3}$$

- *If $d < \frac{a+b}{2}$ then such a strategy inducing an egalitarian type structure forms an ESS (and thus also an NSS) of the meta game.*
- *If $d > \frac{a+b}{2}$ then such a strategy inducing an egalitarian type structure is not a NSS (and thus also not an ESS) of the meta game.*

The proofs of Proposition 2 and 3 are relegated to Appendices A.9 and A.10. They both follow the same steps. First, we compute the unique symmetric full type support equilibrium (if it exists). Evolutionary stability - or instability - then follows from Lemma 3.

Note that, the egalitarian equilibrium payoff v_n lies strictly between u^* and $\frac{a+b}{2}$. It approximates $\frac{a+b}{2}$ as n gets large - from below if $u^* < \frac{a+b}{2}$. In case of a base game with $u^* > \frac{a+b}{2}$, v_n approaches $\frac{a+b}{2}$ from above, but note that by Lemma 1, part 7, we know that in this case $d > \bar{d} > \frac{a+b}{2}$ and hence, by Lemma 4, this full label support egalitarian equilibrium cannot be a NSS.

Proposition 4. *Let $n \equiv |\Theta| \geq 4$ be an even number. If $d < b$ (i.e. anti-coordination base game), then there exists a strategy of the meta-game of any hawk-dove base-game that induces an approximate egalitarian type structure and has full label support.*

For conflict games and for $|\Theta| = 4$ and for $|\Theta| = 6$ we can show that the meta game has no ESS. We do not yet know if this is true for all even $|\Theta|$.

3.4 The meta game with 4 types

Next consider the case $|\Theta| = 4$. With four types there is again the hierarchical structure. Since the number of types is odd, there is no egalitarian structure, but there is a circular structure that is approximate egalitarian.

$$
\begin{array}{c|cccc}
 & L_1 & L_2 & L_3 & L_4 \\
\hline
L_1 & u^* & a & a & a \\
L_2 & b & u^* & a & a \\
L_3 & b & b & u^* & a \\
L_4 & b & b & b & u^*
\end{array}
\qquad
\begin{array}{c|cccc}
 & L_1 & L_2 & L_3 & L_4 \\
\hline
L_1 & u^* & a & a & b \\
L_2 & b & u^* & a & a \\
L_3 & b & b & u^* & a \\
L_4 & a & b & b & u^*
\end{array}
$$

hierarchical   approximate egalitarian

It will turn out to be useful to consider a further reduces form type game in which some labels are summarized in sub-groups which are treated equally by all other labels. For instance:

$$
\begin{array}{c|ccc}
 & G_{12} & L_3 & L_4 \\
\hline
G_{12} & h_2 & a & a \\
L_3 & b & u^* & a \\
L_4 & b & b & u^*
\end{array}
\qquad
\begin{array}{c|ccc}
 & L_1 & G_{23} & L_4 \\
\hline
L_1 & u^* & a & b \\
G_{23} & b & h_2 & a \\
L_4 & a & b & u^*
\end{array}
$$

hierarchical   approximate egalitarian

For both type structure, full label support equilibria exist only for anti-coordination games, but not for conflict games.

Furthermore, for $|\Theta| \geq 4$ there are also structures with a partial hierarchy among some intra-egalitarian groups. Consider the case $|\Theta| = 4$:

$$
\begin{array}{c|cccc}
 & L_1 & L_2 & L_3 & L_4 \\
\hline
L_1 & u^* & a & a & a \\
L_2 & b & u^* & a & b \\
L_3 & b & b & u^* & a \\
L_4 & b & a & b & u^*
\end{array}
\qquad
\begin{array}{c|cccc}
 & L_1 & L_2 & L_3 & L_4 \\
\hline
L_1 & u^* & a & b & a \\
L_2 & b & u^* & a & a \\
L_3 & a & b & u^* & a \\
L_4 & b & b & b & u^*
\end{array}
$$

Top label and egalitarian-group   Egalitarian-group - bottom label

Considering in the first type-game the types L_1-L_3 as one egalitarian group G_T, and in the second type-game the types L_2-L_4 a further reduction of the type-structures is given by

$$
\begin{array}{c|cc}
 & T & G_B \\
\hline
T & u^* & a \\
G_B & b & v_3
\end{array}
\qquad
\begin{array}{c|cc}
 & G_T & B \\
\hline
G_T & v_3 & a \\
B & b & u^*
\end{array}
$$

Top label and egalitarian-group   Egalitarian-group - bottom label

Analogous pre-stable type structures with a hierarchy between a single type and an egalitarian group of $k \equiv |\Theta| - 1$ labels exist, of course, for any even number of labels $|\Theta|$ and lead to the correspondingly further reduced type-structure:

	T	G_B
T	u^*	a
G_B	b	v_k

Top label and egalitarian-group

	G_T	B
G_T	v_k	a
B	b	u^*

Egalitarian-group - bottom label

Remember that $v_k \in [u^*, \frac{a+b}{2}]$ or $[\frac{a+b}{2}, u^*]$. In conflict games with $u^* \geq b$ it follows for all $k \geq 3$ that $v_k > b$. Hence, the top-label (or the labels of the top egalitarian group in the second reduced type-game) dominates the labels of the bottom egalitarian group (or the bottom label, respectively). In equilibrium all probability weight must therefore be on the top-label, or, respectively, on the labels of the top egalitarian group. In anti-coordination games with $u^* < b$ in the first game there is a full label-support equilibrium with a top label and an egalitarian group at the bottom of the hierarchy. In the second game it depends: For sufficiently small k the expected payoff within the egalitarian group v_k is still smaller than b and there is a full -label-support equilibrium, yet there must be a $\bar{k}$ such that for all $k \geq \bar{k}$ the payoff $v_k \geq b$, the payoff of the top-group dominates the payoff of the bottom label payoff and the bottom label cannot be played in equilibrium.

3.5 Group sub-structures

These arguments can be generalized for more hierarchies among groups with different sub-structures.

Definition 4. *Group sub-structures:*

(a) A pre-stable type structure has a **group sub-structure** *if the set of labels Θ can be partitioned into non-empty sets $\Theta_1, ..., \Theta_M$ with $M < |\Theta|$ such that for all $i, j \in \{1, \ldots, M\}$ holds $x_{\theta_i}(\theta_j) = x_{\theta'_i}(\theta'_j)$ for all $\theta_i, \theta'_i \in \Theta_i$ and $\theta_j, \theta'_j \in \Theta_j$.*

(b) A pre-stable structure has a **hierarchy among groups** *if Θ can be partitioned into two nonempty sets Θ_T and Θ_B such that $x_\theta(\theta') = 1$ for all $\theta \in \Theta_T$ and $\theta' \in \Theta_B$.*

(c) In a pre-stable type structure a label θ_T is called a **top label** *if $x_{\theta_T}(\theta) = 1$ for all $\theta \in \Theta \setminus \{\theta_T\}$ and a* **top label within subgroup** *Θ_g if $x_{\theta_T}(\theta) = 1$ for all $\theta \in \Theta_g \setminus \{\theta_T\}$.*

(c) In a pre-stable type structure a label θ_B is called a **bottom label** *if $x_{\theta_B}(\theta) = 0$ for all $\theta \in \Theta \setminus \{\theta_B\}$ and a* **bottom label within subgroup** *Θ_g if $x_{\theta_B}(\theta) = 0$ for all $\theta \in \Theta_g \setminus \{\theta_B\}$.*

Consider the type game of a pre-stable structure with a group sub-structure. A full support strategy in this type game can only be on equilibrium if within each group the payoffs are equilibrated. More precisely, for any subset of labels $\Theta_j \subset \Theta$ let for all $\theta \in \Theta_j$

$$\sigma|_{\Theta_j}(\theta) = \frac{\sigma(\theta)}{\sum_{\theta' \in \Theta_j} \sigma(\theta')}. \tag{4}$$

Definition 5. *Consider the type game of a pre-stable structure with set of labels Θ and with a group sub-structure $(\Theta_1, \dots, \Theta_M)$.*

(a) Let for each Θ_j the **sub-group type game** $\mathfrak{G}_{\Theta_j}$ *denote the $|\Theta_j| \times |\Theta_j|$ game derived from the full type game by eliminating all rows and all columns for labels $\theta_k \notin \Theta_j$.*

(b) A full support strategy σ of the type game with full set of labels Θ is called **equilibrated within group** Θ_j *if under $\sigma|_{\Theta_j}$ every label $\theta \in \Theta_j$ obtains exactly the same expected payoff w_j in the sub-group type game $\mathfrak{G}_{\Theta_j}$.*

(c) A full support strategy of the type game with full set of labels $\Theta = \biguplus_{j=1}^{M} \Theta_j$ is called **within sub-group equilibrated** *if for every Θ_j with $j \in \{1, \dots, M\}$ it is equilibrated within group Θ_j.*

For a strategy of the meta game σ that induces a pre-stable type structure that is within sub-group equilibrated we can now introduce a further reduced **inter-group type game** that has one pure strategy ϑ_j, $j \in \{1, \dots M\}$, for every sub-group and a $M \times M$ payoff matrix with payoff w_j on the diagonal and payoffs a and b as induced by the original type game. Note further that a strategy σ of the original meta game induces a strategy $\hat{\sigma}$ in the inter-group type game via

$$\hat{\sigma}(\vartheta_j) = \sum_{\theta' \in \Theta_j} \sigma(\theta'). \tag{5}$$

Definition 6. *Consider a full label-support strategy of the meta game with set of labels Θ inducing a type game of a pre-stable structure with a group sub-structure $(\Theta_1, \dots, \Theta_M)$, within sub-group equilibrated. The induced full support strategy σ of the type game with full set of labels Θ is called* **inter-group equilibrated** *if every induced strategy in the inter-group type game earns the same expected payoff.*

Definition 7. *A full support strategy σ of the type game induced by a pre-stable structure is called* **equilibrated** *if every pure strategy in the type game earns the same expected payoff under σ.*

Lemma 8. *Consider a pre-stable type structure with a group sub-structure $\Theta_1, ..., \Theta_M$. A full label support strategy σ of the type game with set of types $\Theta = \biguplus_{j=1}^{M} \Theta_j$ is equilibrated if and only if*

- *it is within sub-group equilibrated, and*

- *it is inter-group equilibrated.*

Note that a full support strategy σ of the type game is equilibrated if and only if it is a Nash equilibrium of the type game. Lemma 8, in conjunction with Lemma 7, gives us therefore a clear picture when when a full label support NSS or ESS exists for a pre-stable structure with sub-group structure.

Proposition 5. *Consider a conflict base-game. For the corresponding meta game with $|\Theta| \geq 2$ no ESS and no full label support NSS can exists with ...*

(a) ... a top label, or

(b) ... a bottom label, or

(c) ... a hierarchy among groups, or

(d) ... a group sub-structure in which one group with more than one label has has a top player (within that group), or

(e) ... a group sub-structure in which one group with more than one label has has a bottom player(within that group).

3.6 Welfare

Lemma 9. *Consider the average payoff in a type game induced by a pre-stable type structure.*

- *If $u^* < \frac{a+b}{2}$ then the average payoff is maximized by an equal distribution over all labels and minimized by having all weight on one label only.*

- *If $u^* > \frac{a+b}{2}$ then the average payoff is maximized by a distribution that has only one type in its support and is minimized by an equal distribution over all labels.*

The next proposition characterizes which NSS and which ESS (if they exist) are efficient among all possible distributions over pre-stable structures and which are at least Pareto dominating any other NSS. Note that in our setting at least one NSS exists always and that $a > \bar{d} > \frac{a+b}{2} > b$ is always guaranteed.

Proposition 6. *Welfare properties of NSS and ESS:*

(a) If $d > \bar{d}$: any NSS has only one label in its support and earns the expected payoff u^*, which is efficient (since $u^* > \frac{a+b}{2}$). No ESS exists.

(b) *If $\bar{d} > d > \frac{a+b}{2}$: still any NSS has only one label in its support and earns the expected payoff u^*, but this is now inefficient (since now $u^* < \frac{a+b}{2}$) and the lowest payoff in any pre-stable equilibrium. Note that $u^* \in (b, \frac{a+b}{2})$. No ESS exists.*

(c) *If $\frac{a+b}{2} > d \geq b$: For $|\Theta| \geq 3$ several NSS with distinct payoffs exist. The NSS with only one label in its support give the minimum payoff among pre-stable equilibria. For odd $|\Theta|$ the egalitarian NSS (which is also ESS) gives the maximum expected payoff. Note that the payoff is in $[u^*, v_n] \subset [b, \frac{a+b}{2})$.*

(d) *If $b > d$: In these anti-coordination games many ESS and even more NSS exist. Egalitarian ESS (which exist for odd $|\Theta|$) give the maximum expected payoff. For even $|\Theta|$ approximate egalitarian ESS exist. Hierarchical ESS also exist and are Pareto dominated by the egaliatrian or approximate egalitarian ESS.*

We conjecture that the hierarchical structure gives the lowest expected payoff among ESS (i.e. the expected payoff of any ESS is in $[h_n, v_n]$, with $h_n < b$, $lim_{n\to\infty}h_n = b$, $v_n \in (u^*, \frac{a+b}{2})$, and $\lim_{n\to\infty} v_n = \frac{a+b}{2}$. Furthermore, we conjecture that a structure with only two labels in its support gives the lowest expected payoff among NSS.

4 Discussion

4.1 Relation and Contribution to the cheap talk literature

4.1.1 Cluster points in payoff space

Banerjee and Weibull (2000) study NSS of the meta-game when the base game is a coordination game. Denote by U_n the set of ex-ante expected payoffs in an NSS of the meta-game when the set of types has n elements. Banerjee and Weibull (2000) show that the union of all these payoffs sets $\bigcup_{n=0}^{\infty} U_n$ has a unique cluster point, which is the Pareto efficient Nash equilibrium payoff.

In contrast, for anti-coordination games, we can show that the set of possible NSS payoffs has multiple cluster points. For instance, as every meta-game has a hierarchical NSS by Proposition 2, there is a cluster point at b (the lower payoff in the asymmetric pure strategy equilibrium of the base game). This follows immediately from Lemma 11. However, every anti-coordination meta-game with an odd number of types has also an egalitarian NSS by Proposition 3, which implies that there is another cluster point at $\frac{a+b}{2}$.

Also, for conflict games with $d < \frac{a+b}{2}$ we have at least two cluster points. One cluster point at $\frac{a+b}{2}$, by Proposition 3, and a cluster point at u^* since we always have a single type NSS in this case.

4.1.2 Connection with Farrell, 1987

Farrell (1987) was, as far as we know, the first who studied a model in which there is cheap talk before a game of anti-coordination is played once. In his model players engage in $T \geq 1$ rounds of communication. At each stage $t \leq T$ both players simultaneously and independently of each other send one of two messages, labelled H and D. Farrell (1987) investigates equilibria of this game, in which play after communication is given by the following rule. The player who sent message H at the first point in time at which both players sent different messages (if there is such a time) then plays action H in the anti-coordination game. The other player the plays action D. If both players send identical messages in every round, then they play the symmetric equilibrium x^*.

More formally, let $\theta = (\theta_t)_{t=1}^T$ be a vector of messages, one message for each point in time. Let Θ be the set of all such vectors. In our language this is a set of types. For each pair of types $\theta, \theta' \in \Theta$ let $t^*(\theta, \theta') = \min_t\{\theta_t \neq \theta'_t\}$. If $\theta_t = \theta'_t$ for all t let $t^*(\theta, \theta') = \infty$.

In the language of this paper, Farrell (1987) investigates equilibria of the meta-game that satisfy

$$x_\theta(\theta') = \begin{cases} x^* & \text{if} \quad t^*(\theta,\theta') = \infty \\ 1 & \text{if} \quad t^*(\theta,\theta') < \infty \text{ and } \theta_{t^*(\theta,\theta')} = H \\ 0 & \text{if} \quad t^*(\theta,\theta') < \infty \text{ and } \theta_{t^*(\theta,\theta')} = D \end{cases} \tag{6}$$

It is straightforward to see that this corresponds to what we here call the "hierarchical" type structure. We can reproduce Farrell's (1987) result by noting that every meta-game with a finite number of types has a (unique - up to relabelling of types) hierarchical NSS. The ex-ante expected payoff in this NSS is bounded from above by b (the lower payoff in the asymmetric pure strategy equilibrium of the anti-coordination game). As T tends to infinity the payoff in this NSS tends to b and is, thus, even in this limit, far away from the efficient payoff of $\frac{a+b}{2}$. Note that all this requires that the game is one of anti-coordination.

For conflict game, we know that there is no hierarchical NSS (or even Nash equilibrium). Imposing the hierarchical structure in these games would yield the result that every hierarchical NSS places probability 1 on a single type. We also know now, however, that there are other NSS, based for instance on the egalitarian structure.

For a final example, to see how the egalitarian structure could be implemented in Farrell's (1987) model, consider the case $T = 2$. We then have four "types" given by $(H, H), (H, D), (D, H), (D, D)$. The egalitarian structure could then be imposed as follows.

	(H,H)	(H,D)	(D,H)	(D,D)
(H,H)	u^*	a	b	a
(H,D)	b	u^*	a	a
(D,H)	a	b	u^*	a
(D,D)	b	b	b	u^*

This type game has an NSS (provided $u^* < \frac{a+b}{2}$), in which the first three types are used with probability $\frac{1}{3}$ each, while type (D,D) is not used.

4.1.3 Connection with Hurkens and Schlag, 2002

Hurkens and Schlag (2002) consider the effect of cheap talk messages on equilibrium in coordination games and - more closely related to our paper - a task allocation game which is a specific anti-coordination game in our terminology. They also conjecture that their results hold for a class of games that corresponds to anti-coordination games, a conjecture that our analysis confirms. Specifically, they are interested in the effect of an option not to take part in cheap talk communication, which they model as a special cheap talk message "stay away from cheap talk" which commits the sender to play one action of the base game without conditioning on the opponent's message. For coordination games Hurkens and Schlag (2002) find that without the option to stay away from cheap talk there exists an inefficient evolutionary stable strategy, but with the option to stay away from cheap talk, the set of strategies resulting in the efficient outcome is the unique evolutionary stable set. Most related to our paper is their analysis of the task allocation game without the option to stay away (Section 4.1): We originally worked only with NSS and Lemma 2, until we became aware of the connection to their results. The necessary conditions for ESS that they established in their Lemma 2 inspired our characterization of ESS (Lemma 3 in our paper). In their proof of Proposition 3 they construct for their task allocation game evolutionary stable strategies that correspond to our hierarchical type structure and to our egalitarian or approximate egalitarian type structure, respectively. They continue their analysis of the task allocation game by adding again the option to stay away from communication (Section 4.2) and show that while (in our terminology) the hierarchical type structure equilibrium remains evolutionary stable, all evolutionary stable strategies are bounded away from the efficient outcome (and hence the egalitarian type structure is not evolutionary stable anymore). While the option to stay away from pre-play communication seems plausible in the cheap talk context, in our context where players meet automatically and can condition their play on visible features (labels) of the opponent it seems difficult to visibly commit not to do so.

4.2 Adding one more type

Throughout this paper so far, we always focussed on a finite set of possible types. This means, in particular, for full type support NSS, that there are no unused types. But then the restriction to a fixed set of types seems somewhat arbitrary. In this section we discuss the possible ways one can think about what could happen if one additional "radically new" type suddenly appeared. Suppose we have a finite set of types Θ and evolution has progressed to the point that an NSS of the meta-game has established itself. Now suppose a, previously unheard of, type $\theta^* \notin \Theta$ appears.

There are many ways one could think about what could happen next, but we feel it reasonable to assume that the presence of this new type, now available to be adopted by individuals, will not upset the type structure of the incumbent types. Suppose, for instance, the NSS of the original game (without type θ^*) has a full egalitarian structure. Let us assume that the introduction of the new type does not change that. Having assumed that we now have to think about what behavior the old types will display when they meet the new type and, conversely, what behavior the new type will display when meeting other types.

One way to think about this is to assume nothing other than evolution will now lead to some new NSS, in which the old types interact with each other as before, but any stable behavior between the new and old types can emerge. If this is our view, then there is an even sharper distinction between anti-coordination games and conflict games.

In anti-coordination games, the new NSS (with the given restriction) may possibly look quite different from the old NSS, but we know that any new NSS must still have at least two types in its support. So the multiplicity of types is in this sense stable or robust to the introduction of a radically new type.

This is not true for conflict games (and here it does not matter whether $u^* < \frac{a+b}{2}$ or not). For conflict games the new type evolve to be such that it plays hawk (H) against all other types and they play dove D against it. In this case, however, this new type dominates all other types, and the only NSS with this type structure is the one in which the new type receives probability weight one. In this sense, the possible multiplicity of types in conflict games is not stable or robust to the introduction of a radically new type.

Going back to anti-coordination games, it is interesting to note, that, while the multiplicity of types is robust, the NSS can nevertheless change dramatically from before to after the introduction of the new type. To see this consider the three-type hierarchical structure with one added type X as given below

	T	M	B	X
T	u^*	a	a	b
M	b	u^*	a	a
B	b	b	u^*	b
X	a	b	a	u^*

The type game without type X has a unique NSS, and that NSS has full type support. The meta game with type X has an egalitarian NSS with equal support on T,M, and X, while B is not in its support. This is true for conflict games (provided $u^* < \frac{a+b}{2}$), but more importantly also for anti-coordination games as long as $\frac{1}{3}(u^* + a + b) > b$. This is for instance true when $c = d = 0$ and $b = 1$ and $a = 3$.

5 Conclusion

We investigated the evolution of taking roles in symmetric 2×2 games with asymmetric pure strategy equilibria. We provided a characterization of evolutionary (and neutrally) stable strategies in the meta game. Depending on the parameters and the number of payoff-irrelevant labels or roles we discussed social structures that can emerge as evolutionary (and neutrally) stable strategies and their welfare implications. Two structures of particular interest are the egalitarian and hierarchical type structure. The results are very different for two sub-classes of these base games: conflict games and anti-coordination games. In situations in which different payoffs can be sustained by neutrally stable strategies, the payoffs in the egalitarian type structure Pareto dominate the payoffs of the hierarchical types structure. In this sense an egalitarian organized social structure can promote efficiency in our setting.

It remains an interesting question which neutrally (or evolutionary) stable strategies are likely to emerge if several exist. One way to think about it is to consider further equilibrium refinement. Hierarchical social structures seem fairly common and typically a structure that is cognitively easier to process. If we would assume in addition that players have small cognitive costs for perceiving different labels as separate, this may influence which equilibria remain stable and seems an interesting road for further inquiry. Another way to deal with multiple stable equilibria is to embrace them: It is in the case of multiple stable equilibria that our theoretical analysis and recommendations might have an impact by changing from one focal point equilibrium to a new (and hopefully better) one.[18] Thus, economic theorist might see multiple stable equilibria as a blessing rather than a curse.

[18]For instance, Roger Myerson pointed out in his lecture at GAMES 2016 that much of economic activity (such as exchanging goods for a piece of paper called money) might be best interpreted as a shift from one focal equilibrium to another.

A Proofs

A.1 Proof of Lemma 1

The payoff u of Player 1 in the base game if he plays hawk with probability $x \in [0,1]$ and Player 2 plays hawk with probability $y \in [0,1]$ is given by

$$\begin{aligned} u(x,y) &= xyc + x(1-y)a + (1-x)yb + (1-x)(1-y)d \\ &= d + y(b-d) + x\left[a - d - y\left(a - d + b - c\right)\right]. \end{aligned} \tag{7}$$

If the opponent plays $y^* = \frac{a-d}{a-d+b-c}$ then the term in square brackets is zero, Player 1's expected payoff is $u^* = d + y^*(b-d) = \frac{ab-cd}{a-d+b-c}$, independently of his own action. Player 1 is thus willing to mix and (x^*, x^*) is the mixed equilibrium. If $y > \frac{a-d}{a-d+b-c}$, then the term in square brackets is negative and $x = 0$ is the unique best response. If $y < \frac{a-d}{a-d+b-c}$, then the term in square brackets is positive and $x = 1$ is the unique best response. Hence, in addition to the mixed equilibrium there are exactly two more Nash equilibria: $(x = 0, y = 1)$ and $(x = 1, y = 0)$. This proves the first three points.

To prove point 4 consider the equivalence of the following inequalities.

$$\begin{aligned} & \frac{ab-cd}{a-d+b-c} < b, \\ \Leftrightarrow\ & -cd < -bd + b^2 - cb, \\ \Leftrightarrow\ & d(b-c) < b(b-c), \\ \Leftrightarrow\ & d < b. \end{aligned}$$

Now we prove point 5: For conflict games ($b \leq d$) we now from the previous point that $b \leq u^*$ and hence $b = \min\{b, u^*\}$. The opponent can limit the payoff of a player to b by playing H (hawk). (Each player can also guarantee himself a payoff of at least b by playing strategy D. Hence, a player's minmax value is indeed b for conflict games.)

For anti-coordination games ($b > d$) we now from the previous point that $b > u^*$ and hence $u^* = \min\{b, u^*\}$. The opponent can limit the expected payoff of a player to u^* by playing x^*.

To prove point 6 consider the function $u^* : (-\infty, a) \to \mathbb{R}$ defined by $u^*(d) = \frac{ab-cd}{a-c+b-d}$ for fixed parameter values a, b, c. Then the first derivative of this function is strictly positive:

$$\left(u^*\left(d\right)\right)' \quad = \quad \frac{\left(-c\right)\left(a-d+b-c\right) - \left(ab-cd\right)\left(-1\right)}{\left(a-d+b-c\right)^2} = \frac{\left(a-c\right)\left(b-c\right)}{\left(a-d+b-c\right)^2} > 0.$$

Hence u^* is strictly increasing in d. Furthermore,

$$\lim_{d\to-\infty} u^*(d) = \lim_{d\to-\infty} \frac{ab}{a+b-c-d} - c\frac{1}{\frac{a+b-c}{d} - 1} = 0 + (-c)\frac{1}{0-1} = c, \tag{8}$$

and, by continuity of u^*

$$\lim_{d \to a} u^*(d) = u^*(d = a) = \frac{ab - ca}{a - a + b - c} = \frac{a\,(b - c)}{b - c} = a. \tag{9}$$

To prove point 7 note that

$$\begin{aligned} & u^* = \frac{ab - cd}{a - d + b - c} > \frac{a + b}{2} \\ \Leftrightarrow\ & 2(ab - cd) > (a + b)(a - d + b - c) \\ \Leftrightarrow\ & da + db - 2dc > a^2 + b^2 - ac - bc \\ \Leftrightarrow\ & d > \frac{a^2 + b^2 - c(a + b)}{a + b - 2c}. \end{aligned}$$

We, thus, have

$$\bar{d} = \frac{a^2 + b^2 - c\,(a + b)}{a + b - 2c}. \tag{10}$$

Next, we show $\bar{d} \in (\frac{a+b}{2}, a)$:

$$\begin{aligned} & \bar{d} && > \frac{a + b}{2} \\ \Leftrightarrow\ & \tfrac{a^2+b^2-c(a+b)}{a+b-2c} && > \frac{a + b}{2} \\ \Leftrightarrow\ & 2\left(a^2 + b^2 - ac - bc\right) && > (a + b)\,(a + b - 2c) \\ \Leftrightarrow\ & 2a^2 + 2b^2 - 2ac - 2bc && > a^2 + b^2 + 2ab - 2ac - 2bc \\ \Leftrightarrow\ & 2a^2 + 2b^2 - 2ac - 2bc && > a^2 \\ \Leftrightarrow\ & a^2 + b^2 && > 2ab \\ \Leftrightarrow\ & (a - b)^2 && > 0. \end{aligned} \tag{11}$$

The last inequality is obviously true, which implies the first inequality.

Furthermore, by assumption we have $a > b$ and $b > c$ which implies

$$\begin{aligned} & a(b - c) > b(b - c) \\ \Leftrightarrow\ & 0 > b^2 + ac - cb - ab \\ \Leftrightarrow\ & a^2 + ab - 2ac > a^2 + b^2 - ca - cb \\ \Leftrightarrow\ & a > \frac{a^2 + b^2 - c(a + b)}{a + b - 2c} \\ \Leftrightarrow\ & a > \bar{d}. \end{aligned}$$

This completes the proof of point 7.

A.2 Proof of Lemma 2

We need to prove that any $\sigma \in \Delta(S)$ that does not satisfy conditions (a) and (b) is not an NSS. To prove part (a) consider a $\sigma \in \Delta(S)$ such that there is a $\theta \in \Theta$ with $\sigma(\theta) > 0$ and $x_\theta(\theta) > x^*$ (the case $x_\theta(\theta) < x^*$ can be proven analogously), where x^* is the symmetric equilibrium probability of H in the base game (see Lemma 1.2). Now consider a strategy $\sigma' \in \Delta(S)$ with the property that $\sigma'(\theta') = \sigma(\theta')$ for all $\theta' \in \Theta$ and $x'_{\theta'}(\theta'') = x_{\theta'}(\theta'')$ for all $\theta', \theta'' \in \Theta$ such that at least one of θ', θ'' is not equal to θ, and finally $x_\theta(\theta) = 0$. In words, strategy σ' mimics strategy σ in all respects except when adopting type θ and meeting type θ it plays D.[19] Strategy σ' thus generates different payoff than strategy σ against σ only when both strategies adopt type θ (which happens with positive probability). Then, however, strategy σ' describes the unique best response and, thus, generates a higher payoff than strategy σ does (as σ does not prescribe this best response in this case). This violates the FOC of neutral stability and proves part (a).

To prove part (b) consider a strategy $\sigma \in \Delta(S)$ such that there are $\theta, \theta' \in \Theta$ with $\sigma(\theta) > 0$ and $\sigma(\theta') > 0$. A similar argument to the one above that proves part (a) implies that each of the two types must play a best response to the other type, otherwise the FOC of neutral stability is violated. It remains to be shown that the two types playing the symmetric equilibrium of the base game against each other can not be part of an NSS either. Thus, suppose $x_\theta(\theta') = x_{\theta'}(\theta) = x^*$. Then consider strategy $\sigma' \in \Delta(S)$ such that σ' mimics σ in all respects except in its prescription for $x'_\theta(\theta')$ and $x'_{\theta'}(\theta)$. In fact let $x'_\theta(\theta') = 1$ and $x'_{\theta'}(\theta) = 0$. It is easy to see that $u(\sigma', \sigma) = u(\sigma, \sigma)$ as the only difference that could occur is when types θ and θ' are employed and then, as σ prescribes the mixed strategy equilibrium strategy x^* both pure actions of the base game H and D give equally payoff against x^*. Thus the FOC for neutral stability is satisfied with equality. We then need to check the SOC and compare $u(\sigma', \sigma')$ with $u(\sigma, \sigma')$. We find that the follow inequalities are equivalent

$$\begin{aligned}
u(\sigma', \sigma') &> u(\sigma, \sigma') \\
\sigma(\theta)\sigma(\theta')a + \sigma(\theta')\sigma(\theta)b &> \sigma(\theta)\sigma(\theta')\left(ax^* + d(1-x^*)\right) + \sigma(\theta')\sigma(\theta)\left(ax^* + d(1-x^*)\right) \\
\sigma(\theta)\sigma(\theta')\left[a+b\right] &> \sigma(\theta)\sigma(\theta')\left[(c+a)x^* + (b+d)(1-x^*)\right] \\
a+b &> (c+a)x^* + (b+d)(1-x^*) \\
a+b &> \frac{(c+a)(a-d)}{a-d+b-c} + \frac{(b+d)(b-c)}{a-d+b-c} \\
a(b-c) + b(a-d) &> c(a-d) + d(b-c),
\end{aligned}$$

where the final inequality is true for all hawk-dove games as, by assumption,

[19] It is easy to see that such a strategy exists. It may not be unique. See footnote 9.

$a > d$ and $b > c$. Thus, the SOC for neutral stability is not satisfied and we arrive at a contradiction, proving part (b).

A.3 Proof of Lemma 3

First we prove that Conditions (a)-(d) are necessary for σ being an ESS. We start by proving that σ can only be an ESS if all labels are played with positive probability (Condition (c)). Suppose to the contrary that there exists a label $\hat{\theta} \in \Theta$ with $\sigma(\hat{\theta}) = 0$. Then consider the strategy σ' which is identical to σ except (potentially) when playing against the label $\hat{\theta}$ (which does not happen in equilibrium). If playing against an opponent with label $\hat{\theta}$, strategy σ' would plays D (dove). Clearly, $u(\sigma', \sigma) = u(\sigma, \sigma) = u(\sigma', \sigma') = u(\sigma, \sigma')$ and hence σ can only be ESS if it happens to be identical to σ'. But then the strategy s playing $\hat{\theta}$ with probability one and playing H (hawk) against all types in the support of σ would obtain a payoff $u(s, \sigma) = a > u(\sigma, \sigma)$ which contradicts that σ was ESS.

Given Condition (c) has to hold in any ESS the necessity of Condition (a) and Condition (b) follow immediately from Lemma 2. The necessity of Condition (d) follows directly from the first order condition of the definition of an ESS. Otherwise σ could not even form a Nash equilibrium with itself.

Now we start proving that Conditions (a)-(d) are sufficient to establish that σ is an ESS of the meta game of the hawk dove base game, provided $\frac{a+b}{2} > d$. First note that under conditions (a) and (b) if $\sigma(\theta)$ is specified for all $\theta \in \Theta$ and $x_\theta(\theta') \in \{0, 1\}$ for all $\theta' \neq \theta \in \Theta$ then σ is uniquely determined. In particular, there is no further equivalent strategy since for any match of different labels a pure (contingent) strategy is played. Note that there are $2|\Theta|$ pure best responses to any σ that satisfies conditions (a)-(d), and that these are all in the support of such a σ. (For each label $\theta \in \Theta$ there are two corresponding pure best replies to σ: Select label θ, play against other types $\theta' \neq \theta$ whatever $x_\theta(\theta')$ the strategy σ prescribes, and play against your own label θ either "H" or "D".) A symmetric Nash equilibrium (σ, σ) is called quasi-strict, if σ has all pure best responses to σ in its support.

For a quasi-strict Nash equilibrium (σ, σ), strategy σ is an ESS if and only if the payoff matrix is negative definite with respect to the support of σ. (see van Damme (1991), Theorem 9.2.7 and the preceding text on pages 220/221).[20]

A $K \times K$ payoff matrix M is called negative definite with respect to a nonempty subset S if and only if

(12)
$$\mathbf{y}^T M \mathbf{y} < 0 \textit{ for all } \mathbf{y} \in \mathbb{R}^K \textit{ with } \mathbf{y} \neq 0, \sum_i y_i = 0, \textit{ and } z_i = 0 \textit{ for } i \notin S.$$

[20]Van Damme attributes Theorem 9.2.7 to Haigh (1975) and Abakus (1980). Similar arguments are used in the proofs of Hurkens and Schlag (2002) and inspired our strategy of proof.

Let M be the $2|\Theta| \times 2|\Theta|$ payoff matrix when we restrict the set of pure strategies to the pure best responses to σ. Let the first $|\Theta|$ pure strategies be those in which "H" is played against an opponent with the same label, and strategies from $|\Theta|+1$ to $2|\Theta|$ those in which "D" is played against an opponent with the same label. On the diagonal this matrix M has then first $|\Theta|$ times the entry c and then $|\Theta|$ times the entry d. Off the diagonal it has half of the entries a and half of the entries b, such that $M_{ij} + M_{ji} = a + b$ for all $i \neq j$. Hence, for any $\mathbf{y} \in \mathbb{R}^{2|\Theta|}$ with $\mathbf{y} \neq 0$, $\sum_{i=1}^{2|\Theta|} y_i = 0$ we have

$$\begin{aligned} \mathbf{y}^T M \mathbf{y} &= c \sum_{i=1}^{|\Theta|} y_i^2 + b \sum_{i=|\Theta|+1}^{2|\Theta|} y_i^2 + \frac{a+b}{2} \sum_{i=1}^{2|\Theta|} \sum_{j=1, j\neq i}^{2|\Theta|} y_i y_j \\ &= \left(c - \frac{a+b}{2}\right) \sum_{i=1}^{|\Theta|} y_i^2 + \left(d - \frac{a+b}{2}\right) \sum_{i=|\Theta|+1}^{2|\Theta|} y_i^2, \end{aligned} \tag{13}$$

where we used $\sum_i \sum_{j\neq i} y_i y_j = \sum_i y_i \left(\sum_{j \neq i} y_j\right) = \sum_i y_i(-y_i) = -\sum_i y_i^2$ to obtain the last line. The first term is always negative because of $a > b > c$. The second term is negative for $\frac{a+b}{2} > d$, which then implies $\mathbf{y}^T M \mathbf{y} < 0$ for any $\mathbf{y} \in \mathbb{R}^{2|\Theta|}$ with $\mathbf{y} \neq 0$, $\sum_{i=1}^{2|\Theta|} y_i = 0$. This implies that the payoff matrix is negative definite with respect to its carrier and together with the fact that (σ, σ) is quasi-strict, it implies that σ is an ESS.

Finally, we prove that $\frac{a+b}{2} > d$ is also a necessary condition for ESS. If, in contrast, $\frac{a+b}{2} \leq d$ then we can choose a vector $\mathbf{y} \in \mathbb{R}^{2|\Theta|}$ that has zeros in the first $|\Theta|$ entries and some non-zero entries in the remaining entries. Then $\mathbf{y}^T M \mathbf{y} \geq 0$ and the corresponding σ cannot be an ESS.

A.4 Proof of Lemma 4

First note that under conditions (a) and (b) if $\sigma(\theta)$ is specified for all $\theta \in \Theta$ and $x_\theta(\theta') \in \{0, 1\}$ for all $\theta' \neq \theta \in \Theta$ then σ is uniquely determined. In particular, there is no further equivalent strategy since for any match of different labels a pure (contingent) strategy is played. Note that there are $2|\Theta|$ pure best responses to any σ that satisfies conditions (a)-(d), and that these are all in the support of such a σ. (For each label $\theta \in \Theta$ there are two corresponding pure best replies to σ: Select label θ, play against other types $\theta' \neq \theta$ whatever $x_\theta(\theta')$ the strategy σ prescribes, and play against your own label θ either "H" or "D".) A symmetric Nash equilibrium (σ, σ) is called quasi-strict, if σ has all pure best responses to σ in its support. Hence, if σ satisfies Conditions (a)-(c), then (σ, σ) is a quasi-strict, symmetric Nash-equilibrium.

For quasi-strict symmetric Nash-equilibria Theorem 9.2.7 in van Damme (1991) provides a nice characterization for ESS which we adapt in the next lemma to NSS.

Lemma 10. *Let (σ,σ) be a quasi-strict Nash equilibrium, i.e. the set of pure best responses $B(\sigma)$ corresponds to the support of σ, $supp(\sigma)$. Define $K \equiv |supp(\sigma)| = |B(\sigma)|$ and let M denote the $K \times K$ matrix corresponding to the restriction of the full payoff to pure strategies in $supp(\sigma)$. Then σ is an NSS if and only if*

$$\mathbf{y}^T M\mathbf{y} \leq 0 \text{ for all } \mathbf{y} \in \mathbb{R}^K \text{ with } \sum_i y_i = 0. \tag{14}$$

Proof of Lemma 10: Now, we show for quasi-strict (σ,σ), that σ NSS implies Condition 14. Let σ be the K-dimensional restriction of σ to the pure strategies in its support. Quasi-strictness of (σ,σ) implies for all $\mu \in \Delta(supp(\sigma))$: $\mu^T M\sigma = \sigma^T M\sigma$. Furthermore, since the FOC holds with equality, the SOC for NSS implies for all $\mu \in \Delta(supp(\sigma))$:

$$\begin{aligned} & \mu^T M\mu && \leq \sigma^T M\mu \\ \Leftrightarrow\ & \mu^T M\mu - \mu^T M\sigma + \sigma^T M\sigma && \leq \sigma^T M\mu \\ \Leftrightarrow\ & (\mu-\sigma)^T M(\mu-\sigma) && \leq 0. \end{aligned} \tag{15}$$

Now suppose, with the aim to construct a contradiction, that $\exists \mathbf{y} \in \mathbb{R}^K$ with $\sum_i y_i = 0$ such that

$$\mathbf{y}^T M\mathbf{y} > 0. \tag{16}$$

Then we can construct a $\mu \in \Delta(supp(\sigma))$ that violates Inequality 15 in the following way: First, define $\epsilon \equiv min\{(\min_i \sigma_i), (\min_i(1-\sigma_i))\}$, and $y_{max} \equiv \max_i |y_i|$ and then set $\tilde{y}_i \equiv \frac{\epsilon}{y_{max}} y_i$. Then $\sum_i \tilde{y}_i = 0$ and $\tilde{\mathbf{y}}^T M\tilde{\mathbf{y}} > 0$. If we set $\mu_i \equiv \tilde{y}_i + \sigma_i$ then $\mu_i \in [0,1]$ and $\sum_i \mu_i = 1$, hence $\mu \in \Delta(supp(\sigma))$, and furthermore $(\mu-\sigma)^T M(\mu-\sigma) > 0$, which contradicts Inequality 15.

Now we show that Condition 15 implies for any σ that forms a quasi-strict Nash equilibrium against itself, that σ is NSS. First, consider the case of a mutant strategy $\mu \in \Delta(supp(\sigma))$. For any such μ Condition 15 implies with $y \equiv \mu - \sigma$

$$\begin{aligned} & (\mu-\sigma)^T M(\mu-\sigma) && \leq 0, \\ \Rightarrow\ & \mu^T M\mu - \mu^T M\sigma + \sigma^T M\sigma - \sigma^T M\sigma && \leq 0, \\ \Rightarrow\ & \mu^T M\mu && \leq \sigma^T M\sigma, \end{aligned} \tag{17}$$

where we used that $\mu^T M\sigma = \sigma^T M\sigma$ for $\mu \in \Delta(supp(\sigma))$ if (σ,σ) is a quasi-strict Nash equilibrium. Hence, for $\mu \in \Delta(supp(\sigma))$ the FOC for NSS is satisfied with equality and the SOC is satisfied by Inequality17. In order to complete the proof that σ is a NSS, note that for any mutant strategy $\mu \notin \Delta(supp(\sigma))$ it follows from the assumption that (σ,σ) is a quasi-strict

Nash-equilibrium, that $\mu^T M\sigma < \sigma^T M\sigma$. Hence, the FOC for NSS is strictly satisfied and the SOC therefore irrelevant. This completes the proof of Lemma 10.

We now continue with the proof of Lemma 4: M is the $2|\Theta| \times 2|\Theta|$ payoff matrix when we restrict the set of pure strategies to the pure best responses to σ. Let the first $|\Theta|$ pure strategies be those in which "H" is played against an opponent with the same label, and strategies from $|\Theta|+1$ to $2|\Theta|$ those in which "D" is played against an opponent with the same label. On the diagonal this matrix M has then first $|\Theta|$ times the entry c and then $|\Theta|$ times the entry d. Off the diagonal it has half of the entries a and half of the entries b, such that $M_{ij} + M_{ji} = a + b$ for all $i \neq j$. Hence, for any $\mathbf{y} \in \mathbb{R}^{2|\Theta|}$ with $\sum_{i=1}^{2|\Theta|} y_i = 0$ we have

$$\begin{aligned} \mathbf{y}^T M\mathbf{y} &= c\sum_{i=1}^{|\Theta|} y_i^2 + d\sum_{i=|\Theta|+1}^{2|\Theta|} y_i^2 + \frac{a+b}{2}\sum_{i=1}^{2|\Theta|}\sum_{j=1,j\neq i}^{2|\Theta|} y_i y_j \\ &= \left(c - \frac{a+b}{2}\right)\sum_{i=1}^{|\Theta|} y_i^2 + \left(d - \frac{a+b}{2}\right)\sum_{i=|\Theta|+1}^{2|\Theta|} y_i^2, \end{aligned} \tag{18}$$

where we used $\sum_i \sum_{j\neq i} y_i y_j = \sum_i y_i \left(\sum_{j\neq i} y_j\right) = \sum_i y_i(-y_i) = -\sum_i y_i^2$ to obtain the last line. The first term is always negative because of $a > b > c$. The second term is non-positive for $\frac{a+b}{2} \geq d$, which then implies $\mathbf{y}^T M\mathbf{y} \leq 0$ for any $\mathbf{y} \in \mathbb{R}^{2|\Theta|}$ with $\sum_{i=1}^{2|\Theta|} y_i = 0$. Together with the fact that (σ, σ) is quasi-strict, Lemma 10 implies that σ is an NSS. If, in contrast, $\frac{a+b}{2} < d$ then, for $|\Theta| \geq 2$, we can choose a vector $\mathbf{y} \in \mathbb{R}^{2|\Theta|}$ that has zeros in the first $|\Theta|$ entries and some non-zero entries in the remaining entries. Then $\mathbf{y}^T M\mathbf{y} > 0$ and the corresponding σ cannot be an NSS. This completes the proof of Lemma 4.

A.5 Proof of Lemma 5

Suppose not. Then there exists a mutant strategy $\mu|\Theta_S$ in the restricted meta game such that violates either the FOC or SOC of NSS in the restricted meta game. The same strategy extended to the full meta game with full set of labels Θ must violate the same NSS condition in the meta game, since all extra labels are played with probability 0 and do not change expected payoffs. Hence any NSS of the meta game, must also form an NSS in the meta game restricted to labels in the support of its strategy, which proves Lemma 5.

A.6 Proof of Lemma 6

A strategy that supports the NSS in the meta game with the larger set of labels Θ is a straightforward extension of the full label support NSS strategy from the meta game with smaller set of labels Θ_S by specifying that $x_\theta(\theta')$ corresponds to the strategy of the base game that gives the opponent only his minmax value: $\min\{u^*, b\}$. Note that the expected payoff (call it v) of any strategy in the support of an NSS with $|\Theta_S| \geq 2$ is strictly above this minmax value: For $u^* \neq b$ and any $\theta \in \Theta_S$: $v \geq \sigma(\theta)u^* + (1-\sigma(\theta))b > \min\{u^*, b\}$. For $u^* = b$, no label in the support of a NSS strategy can play always dove against all other labels in the support (otherwise it is dominated by any of the other strategies in the support, as these obtain sometimes $a > b$ and never below b). Hence there is a label θ' with $\sigma(\theta') > 0$ such that $v \geq \sigma(\theta')a + (1-\sigma(\theta'))b > b = \min\{u^*, b\}$. This proves Lemma 6.

A.7 Proof of Lemma 7

The proof follows immediately from Lemmata 3 and 4.

A.8 Proof of Proposition 1

We shall prove parts (a) and (b) simultaneously. We first prove that a strategy $\sigma \in \Delta(S)$ with both types in its support cannot be an NSS if the base game is an conflict game (i.e. if $b < d$) and that there is an NSS among these strategies with both types in their support if the base game is an anti-coordination game (i.e. if $b > d$) and that it has the properties stated in (b). Call this statement (1).

We then prove that a strategy $\sigma \in \Delta(S)$ with a single type in its support cannot be an NSS if the base game is an anti-coordination game (i.e. if $b > d$) and that there are NSS among these strategies with a single type in their support if the base game is an conflict game (i.e. if $b < d$) and that they have the properties stated in (a). Call this statement (2).

Statements (1) and (2) together are equivalent to Proposition 1 parts (a) and (b) together, excepts for the statements about ESS. But these follow directly from the previous lemmata: In part (a), since any ESS has to be NSS and hast to have full support by Lemma 3, there cannot exist an ESS. In part (b), since in any anti-coordination game the condition $\frac{a+b}{2} > b > d$ is satisfied Lemma 3 shows that the full support NSS strategy must also be an ESS.

Proof of statement (1):

Consider an NSS strategy $\sigma \in \Delta(S)$ such that $\sigma(T) > 0$ and $\sigma(B) > 0$. Then by Lemma 2 we must have $x_T(T) = x_B(B) = x^*$ and, w.l.o.g. $x_T(B) = 1$ and $x_B(T) = 0$. Now consider strategy $\sigma' \in \Delta(S)$ such that $x'_\theta(\theta') = x_\theta(\theta')$ for all $\theta, \theta' \in \Theta$ and $\sigma(T) = 1$. Then $u(\sigma, \sigma) = \sigma(T)\,(\sigma(T)u^* + \sigma(B)b) + \sigma(B)\,(\sigma(T)a + \sigma(B)u^*)$, while $u(\sigma', \sigma) = \sigma(T)u^* + \sigma(B)a$. Suppose the base game is an conflict game (i.e. $b < d$). Then by Lemma 1.4 we have $u^* > b$. By assumption a is the highest payoff possible in the base game and we, thus, have $a > u^*$. But then $u(\sigma', \sigma) > u(\sigma, \sigma)$ violating the FOC of neutral stability. Thus σ cannot be an NSS.

Suppose now that the base game is an anti-coordination game (i.e. $b > d$). Then by Lemma 1.4 we have $u^* < b$. This implies that for $\sigma(T)$ close to zero the best response is to adopt type B, while for $\sigma(T)$ close to 1, the best response is to adopt type T. Thus, there must be an equilibrating $\sigma^*(T)$, which is then an NSS by the same argument that the base game has a (symmetric mixed strategy) NSS. It is easy to verify that $\sigma^*(T)$ is as stated in Part (b) of Proposition 1.

Proof of statement (2):

Consider an NSS strategy $\sigma \in \Delta(S)$ such that there is a type $\theta \in \Theta$ with $\sigma(\theta) = 1$. Let w.l.o.g. $\sigma(T) = 1$. By Lemma 2 we must have $x_T(T) = x^*$.

Note first that any $\sigma' \in \Delta(S)$ with $\sigma'(T) = 1$ satisfies $u(\sigma', \sigma') = u(\sigma, \sigma') = u(\sigma, \sigma)$ and, thus for such σ' the FOC for neutral stability is satisfied with equality and the SOC for neutral stability is satisfied with equality. Note here that the SOC for evolutionary stability is not satisfied, proving this part of the statement of part (a): a strategy with a single type in its support cannot be an ESS.

We, thus, need to consider $\sigma' \in \Delta(S)$ such that $\sigma'(B) > 0$. By the linearity of the payoff function (in the probabilities) it suffices to then consider $\sigma' \in \Delta(S)$ such that $\sigma'(B) = 1$. The optimal σ' among those with $\sigma'(B) = 1$ must play a best response against $x_T(B)$.

Suppose $x_T(B) < x^*$. Then the optimal best response σ' among those with $\sigma'(B) = 1$ is to play H, i.e. to set $x'_B(T) = 1$. In this case $u(\sigma', \sigma) = cx_T(B) + a(1 - x_T(B)))$, while $u(\sigma, \sigma) = u^*$. Note that $u^* = cx^* + a(1 - x^*)$. As $a > c$ by assumption and $x_T(B) < x^*$ we have $u(\sigma', \sigma) > u(\sigma, \sigma)$. This is true for all hawk-dove games (i.e. irrespective of the relationship between b and d). Thus, σ with $x_T(B) < x^*$ violates the FOC of neutral stability and cannot be an NSS.

Suppose next that $x_T(B) > x^*$. Then the best response is to play D, i.e. to set $x'_B(T) = 0$. In this case $u(\sigma', \sigma) = bx_T(B) + d(1 - x_T(B)))$, while $u(\sigma, \sigma) = u^*$. Note that $u^* = bx^* + d(1 - x^*)$. Consider the case $b > d$ (anti-coordination games). As $x_T(B) > x^*$ we then have $u(\sigma', \sigma) > u(\sigma, \sigma)$. Thus for the anti-coordination case σ with $x_T(B) > x^*$ is also not an NSS (as the FOC is violated). Consider the case $b < d$. As $x_T(B) > x$ we then have $u(\sigma', \sigma) < u(\sigma, \sigma)$. Therefore σ satisfies both FOC and SOC of neutral stability, and is thus an NSS.

The only remaining case is $x_T(B) = x^*$. Then any $\sigma' \in \Delta(S)$ is a best response. I.e. we have $u(\sigma', \sigma) = u(\sigma, \sigma)$. We, thus, need to check the SOC for all σ'. As we shall prove that the SOC is not satisfied (irrespective of how b compares with d) we only need to find one σ' that satisfies $u(\sigma', \sigma') > u(\sigma, \sigma')$. The following σ' has this property. Let $\sigma'(T) = \sigma'(B) = \frac{1}{2}$ and

$x'_T(T) = x'_B(B) = x^*$ and $x'_T(B) = 0$ and $x'_B(T) = 1$. Then $u(\sigma', \sigma') = \frac{1}{2}u^* + \frac{1}{4}a + \frac{1}{4}b$, while $u(\sigma, \sigma') = \frac{1}{2}u^* + \frac{1}{2}[x^*c + (1 - x^*)b]$. As $a > c$ by assumption we, thus, have indeed $u(\sigma', \sigma') > u(\sigma, \sigma')$. This finishes the proof.

A.9 Proof of Proposition 2

The following lemma, in conjunction with Lemma 7, immediately proves part (a) of Proposition 2.

Lemma 11. *Let $n \equiv |\Theta| \geq 2$. There exists a full support Nash equilibrium (σ^*, σ^*) of the type-game of the (pre-stable) hierarchical type-structure if and only if the base game is an anti-coordination game ($u^* < b$). If the labels $\theta_1, \theta_2, \ldots, \theta_n$ are ordered according to the hierarchical structure (with θ_1 top type), then for $i \in \{2, \ldots, n\}$:*

$$\begin{aligned}
\sigma^*(\theta_i) &= \sigma^*(\theta_{i-1}) \left(\frac{b - u^*}{a - u^*} \right) = \sigma^*(\theta_1) \left(\frac{b - u^*}{a - u^*} \right)^{i-1} \\
&= \left(\frac{1 - \left(\frac{b-u^*}{a-u^*}\right)}{1 - \left(\frac{b-u^*}{a-u^*}\right)^n} \right) \left(\frac{b - u^*}{a - u^*} \right)^{i-1}, \qquad (19)
\end{aligned}$$

and each type earning the average payoff

$$\begin{aligned}
h_n &\equiv \sigma^*(\theta_1)u^* + (1 - \sigma^*(\theta_1))\, a \\
&= \left(\frac{1 - \left(\frac{b-u^*}{a-u^*}\right)}{1 - \left(\frac{b-u^*}{a-u^*}\right)^n} \right) u^* + \left(\frac{\left(\frac{b-u^*}{a-u^*}\right) - \left(\frac{b-u^*}{a-u^*}\right)^n}{1 - \left(\frac{b-u^*}{a-u^*}\right)^n} \right) a \qquad (20)
\end{aligned}$$

Equivalently,

$$\begin{aligned}
&h_n = \sigma^*(\theta_n)u^* + (1 - \sigma^*(\theta_n))\, b \\
&= \left(\frac{1 - \left(\frac{b-u^*}{a-u^*}\right)}{1 - \left(\frac{b-u^*}{a-u^*}\right)^n} \right) \left(\frac{b - u^*}{a - u^*} \right)^{n-1} u^* + \left(1 - \left(\frac{1 - \left(\frac{b-u^*}{a-u^*}\right)}{1 - \left(\frac{b-u^*}{a-u^*}\right)^n} \right) \left(\frac{b - u^*}{a - u^*} \right)^{n-1} \right) b \\
&= \left(\frac{1 - \left(\frac{b-u^*}{a-u^*}\right)}{1 - \left(\frac{b-u^*}{a-u^*}\right)^n} \right) \left(\frac{b - u^*}{a - u^*} \right)^{n-1} u^* + \left(\frac{1 - \left(\frac{b-u^*}{a-u^*}\right)^{n-1}}{1 - \left(\frac{b-u^*}{a-u^*}\right)^n} \right) b
\end{aligned}$$

Note, that (for anti-coordination games) $h_n < b$ and $\lim_{n\to\infty} h_n = b$.

Proof of Lemma 11: For convenience let $\Theta = \{1, 2, ..., n\}$ (with type 1 top-type) and for any mixed strategy $\sigma \in \Delta(S)$ let $\alpha_k \equiv \sigma(k)$. Let $\alpha \in \Delta(S)$ denote the full support symmetric Nash equilibrium. Given α every type k

(fixing the hierarchical type structure) must yield the same expected payoff. The payoff to type k is given by $A_k = \sum_{l=1}^{k-1} \alpha_l b + \alpha_k u^* + \sum_{l=k+1}^{n} \alpha_l a$. Equating A_k and A_{k+1} yields $\alpha_k u^* + \alpha_{k+1} a = \alpha_k b + \alpha_{k+1} u^*$. This, in turn yields $\frac{\alpha_{k+1}}{\alpha_k} = \frac{b-u^*}{a-u^*}$, which must be true for all $k \in \{1, ..., n-1\}$. This corresponds to the first equality. This is only possible with full support if $b > u^*$ and hence the base game must be an anti-coordination game. If the game in hand is one of anti-coordination, this ratio is a number strictly between 0 and 1. The second equality follows by induction and the third equality from the requirement $1 = \sum_{i=1}^{n} \alpha_i = \alpha_1 \sum_{i=1}^{n} \left(\frac{b-u^*}{a-u^*}\right)^{i-1} = \alpha_1 \left(\frac{1-\left(\frac{b-u^*}{a-u^*}\right)^n}{1-\frac{b-u^*}{a-u^*}}\right)$, where the last step follows from the well known equality $\sum_{i=0}^{N} \delta^i = \frac{1-\delta^{N+1}}{1-\delta}$, which is easily proved by induction over N. This proves Lemma 11.

To prove part (b) of Proposition 2 note first that for anti-coordination games part (b) follows directly from part (a) since every ESS is also NSS. For conflict games the argument in the proof of Lemma 11 shows that any NSS with hierarchical type structure must have all weight on the top label. It remains only to be shown that this is indeed an NSS of the meta game: Any strategy playing any other label with positive probability earns b or less against the incumbent top-label population while incumbents earn $u^* \geq b$. In games $u^* > b$ the mutant earns strictly less in the FOC. In the knife edge case of a base game with $u^* = b$ the FOC is satisfied with equality if dove is played against the top label with certainty, but then the incumbents earn a against the mutants, while mutants earn strictly less than a against themselves.

A.10 Proof of Proposition 3

Under an egalitarian type structure each type plays H against half of all other types and D against the other half. It is easy to see that for odd $|\Theta|$ (then we can find a natural number l such that $|\Theta| = 2l + 1$) there are such egalitarian pre-stable structures, i.e. it is a well defined structure. We can, for instance, locate the $2l + 1$ labels on a circle and each label plays H against the next l labels located clockwise and D against the next l labels located anti-clockwise.

For convenience let $\Theta = \{1, 2, ..., n\}$.Let $\alpha \in \Delta(\Theta)$ denote the full support symmetric Nash equilibrium of the type game given by $\alpha_k = \frac{1}{n}$ for all $k \in \{1, ..., n\}$.

The expected expected payoff of each strategy in the type game against α is given by:

$$v_n \equiv \frac{u^*}{n} + \frac{n-1}{n}\frac{a+b}{2}. \tag{21}$$

It follows immediately from Lemma 7 that for $d < \frac{a+b}{2}$ the corresponding strategy of the meta game forms an ESS, and that $d < \frac{a+b}{2}$ it cannot form an NSS, q.e.d.

A.11 Proof of Proposition 4

If $|\Theta| \geq 4$ is an odd number, we can find a natural number $l \geq 2$ such that $|\Theta| = 2l$. For anti-coordination games, we now construct an approximate egalitarian pre-stable type structure with a full support Nash equilibrium in the type game. (For their task allocation game Hurkens and Schlag (2002) have a similar construction in the proof of their Prop. 3). Imagine the $2l$ labels placed on a circle. Labels $i \in \{1, \ldots, l\}$ play H against the l next labels located clockwise and D against the $l-1$ labels located anti-clockwise. Labels $i \in \{l+1, \ldots, 2l\}$ play H against the $l-1$ next labels located clockwise and D against the l labels located anti-clockwise. This forms an pre-stable structure if all labels play also x^* against their own label.

Consider now the corresponding type game. This has a full support Nash-equilibrium if and only if there is a full support mixed strategy $\alpha = (\alpha_1, \ldots \alpha_{2l}) \in \Delta(\Theta)$ in the type game such that all labels earn the same expected payoff. Hence, the difference between the payoff of any label θ_i and the payoff of the clockwise next label $\theta_{i+1(mod\ 2l)}$ must be zero:
For $1 \leq i < l$:

$$\alpha_i \left(u^* - b\right) + \alpha_{i+1} \left(a - u^*\right) + \alpha_{i+l+1} \left(b - a\right) = 0, \tag{22}$$

for $i = l$:

$$\alpha_l \left(u^* - b\right) + \alpha_{l+1} \left(a - u^*\right) = 0, \tag{23}$$

for $l + 1 \leq i \leq 2l - 1$:

$$\alpha_i \left(u^* - b\right) + \alpha_{i+1} \left(a - u^*\right) + \alpha_{i-l} \left(b - a\right) = 0. \tag{24}$$

(Together, these equations should also automatically imply for $i = 2l$:

$$\alpha_l \left(u^* - b\right) + \alpha_1 \left(a - u^*\right) + \alpha_l \left(b - a\right) + \alpha_{l+1} \left(b - a\right).) \tag{25}$$

Note first, that for conflict games ($u^* \geq b$) the equation $\alpha_l \left(u^* - b\right) + \alpha_{l+1} \left(a - u^*\right) = 0$ has no solution (with $\alpha_l, \alpha_{l+1} \geq 0$). For $|\Theta| = 4$ it is straightforward to show that all approximate egalitarian structures have the structure above and thus no approximate egalitarian structure can be part of an ESS of the meta-game in this case.

Now we prove that there is an ESS of the meta-game under the approximate egalitarian type structure when the base game is an anti-coordination game. We proceed by first establishing a lemma that provides a necessary condition for an arbitrary finite symmetric two player game to have a symmetric completely mixed Nash equilibrium.

A few definitions are necessary. For an $m \times n$ matrix A let $\mathrm{col}(A)$ denote the set of column vectors of A. For any vector $a \in \mathbb{R}^m$ let $\mathrm{HS}(a)$ denote the *half space* induced by a, given by the set of all vectors $v \in \mathbb{R}^m$ such that $v^T a \leq 0$. For an $m \times n$ matrix A let $\mathrm{HS}(A) = \bigcup_{a \in \mathrm{col}(A)} \mathrm{HS}(a)$ denote the union of all half spaces of columns of A. Note that for every element $v \in \mathrm{HS}(A)$ there is a $a \in \mathrm{col}(A)$ such that $v^T a \leq 0$.

For a symmetric finite two player game with $n \times n$ payoff matrix A let $D = D(A)$ denote A-induced payoff difference matrix given by the $n \times n-1$ matrix obtained from A as follows. The k-th row of D is the difference between rows $k+1$ and k, for $k = 1, 2, ..., n-1$. Finally denote by $\bar{D} = \bar{D}(A)$ the $n \times n$ matrix coincides with D for the first $n-1$ rows and has the unit vector (vector of all ones) in row n.

Lemma 12. *Consider a finite symmetric two player normal form game with $n \times n$ payoff matrix A. If this game has a symmetric completely mixed Nash equilibrium then $HS(D(A)) = \mathbb{R}^{n-1}$, i.e. the union of half-spaces induced by the set of columns of the payoff difference matrix covers the whole set $\mathbb{R}^{n-1}$.*

Proof: A necessary condition for this game to have a completely mixed Nash equilibrium is that there is a vector $x \in \mathbb{R}^n$ with $x \geq 0$ (that is each coordinate satisfies $x_i \geq$ and there is one i such that $x_i > 0$) such that $\bar{D}x = b$, where $b = (0, 0, ..., 0, 1)^T \in \mathbb{R}^n$.

By Farkas' lemma this implies that there is no $v = (v_1, v_2, ..., v_n)^T \in \mathbb{R}^n$ with $v^T b > 0$ such that $v^T \bar{D} \leq 0$. Note that the condition $v^T b > 0$ is satisfied if and only if $v_n > 0$. Let $w = w(v)$ be the vector in $\mathbb{R}^{(n-1)}$ that consists of the first $n-1$ coordinates of v. Note that the condition $v^T \bar{D} \leq 0$ is satisfied if and only if $w^T D \leq v_n$. This implies that $w^T D \leq 0$.

Thus a necessary condition for this game to have a completely mixed Nash equilibrium is that there is no $w \in \mathbb{R}^{(n-1)}$ such that $w^T D \leq 0$. This implies that for every $w \in \mathbb{R}^{(n-1)}$ there is a vector $d \in \mathrm{col}(D)$ such that $w^T d > 0$ and, thus, $w \in \mathrm{HS}(D)$. This implies $\mathrm{HS}(D) = \mathbb{R}^{(n-1)}$. QED

Consider the type game with an approximate egalitarian structure as described as above with an even number of types $n = 2k$, for any $k = 1, 2, ...$ The payoff difference matrix D induced by this game is as follows. Column 1 has two non-zero entries, the first in row 1 given by $b - u^*$, the second in row $k+1$ given by $a - b$. Column i with $2 \leq i \leq k-1$ has three non-zero entries at row $i-1$ given by $-(a - u^*)$, at row i given by $b - u^*$, and at row $k+i$ given by $a - b$. Column k has two non-zero entries at row $k-1$ given by $-(a - u^*)$ and at row k given by $b - u^*$. Column $k+1$ has two non-zero entries at row k given by $-(a - u^*)$ and at row $k+1$ given by $b - u^*$. Column i with $k+2 \leq i \leq 2k-1$ has three non-zero entries at row $i - (k+1)$ given by $a - b$, at row $i-1$ given by $-(a - u^*)$, and at row i given by $b - u^*$.

Finally, column $2k$ has two-non-zero entries, one at row $k-1$ given by $a-b$ and one at row $2k-1$ given by $-(a-u^*)$.

Let d_1 denote the first column of this matrix D. Let d^* denote the sum of all columns 2 to n. Let d^{**} denote the sum of all columns 2 to n except column $k+1$. The three vectors can be summarized as follows

$$\begin{array}{c|ccc}
\text{coordinate} & d_1 & d^* & d^{**} \\
1 & b-u^* & -(b-u^*) & -(b-u^*) \\
2 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots \\
k-1 & 0 & 0 & 0 \\
k & 0 & b-u^* & -(a-b) \\
k+1 & a-b & -(a-u^*) & -(a-b) \\
k+2 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots \\
2k-1 & 0 & 0 & 0
\end{array}$$

Next we show that $\mathrm{HS}(D) = \mathbb{R}^{2k-1}$. To see this note that $\mathrm{HS}(d_1) \cup \mathrm{HS}(d^*) \cup \mathrm{HS}(d^{**}) \subset \mathrm{HS}(D)$ given that d^* and d^{**} are positive linear combinations of columns of D. Next note that there is a positive linear combination of d^* and d^{**}, denoted $\hat{d}$, that equals exactly the negative of d_1 (this requires $u^* < b$). We then have that $\mathbb{R}^{2k-1} = \mathrm{HS}(d_1) \cup \mathrm{HS}(\hat{d}) \subset \mathrm{HS}(D)$.

We thus have established the necessary condition of Lemma 12 for the existence of a completely mixed Nash equilibrium in this type game. In fact we have shown that there is a mixed strategy that if one player uses it the other is completely indifferent between all strategies.

Next we need to show that this mixed strategy is completely mixed. Suppose not. Suppose $x \geq 0$ and there is a coordinate i such that $x_i = 0$ and nevertheless $Dx = 0$. Now denote by $\hat{D}$ the matrix obtained from D by removing column i and let $\hat{x}$ be obtained from x by removing coordinate i. If $\hat{D}\hat{x} = 0$ and $x \geq 0$ by Farkas' Lemma we must again have that there is no $w \in \mathbb{R}^{2k-1}$ such that $w^T D \leq 0$. First suppose we remove any column $i \geq 2$. Then $\hat{D}$ has a row with only non-negative entries, denote this row by row j. Then choosing a vector w such that $w_j = -1$ and all other coordinates equal to zero, we have $w^T D \leq 0$, a contradiction. Now suppose we remove column 1. Then the vector w of all ones is such that $w^T D \leq 0$.

A.12 There is no ESS when $|\Theta| = 4$ in the conflict case

Suppose the base game is one of conflict with $\frac{a+b}{2} > d > b$ and $|\Theta| = 4$. We now show that this meta game has no ESS. By Lemma 3 any ESS must have full support on all four types. We then show that any candidate ESS that satisfies properties a and b of Lemma 3, necessarily has a dominated type, and thus cannot have full support, violating property c of Lemma 3.

We have to go through a series of cases. First, suppose that there is one type who plays H against all other types. Then this type dominates all other types, and we arrive at a contradiction. Second, suppose that there is one type who plays D against all other types. Then this type is dominated by all other types, a contradiction. The only case remaining is such that all types play H against at least one other type and at most two other types. This pins down a unique type structure (subject to relabeling), the unique approximate egalitarian structure (subject to relabeling) given by the following matrix.

$$\begin{array}{c|cccc} & L_1 & L_2 & L_3 & L_4 \\ \hline L_1 & u^* & a & a & b \\ L_2 & b & u^* & a & a \\ L_3 & b & b & u^* & a \\ L_4 & a & b & b & u^* \end{array}$$

Given $d > b$ and thus $u^* > b$, type L_3 is dominated by type L_2.

A.13 The only two ESS when $|\Theta| = 5$ in the conflict case

Suppose the base game is one of conflict with $\frac{a+b}{2} > d > b$ and $|\Theta| = 5$. This game has exactly two ESS. One is the egalitarian one. The other is as follows.

$$\begin{array}{c|ccccc} & L_1 & L_2 & L_3 & L_4 & L_5 \\ \hline L_1 & u^* & b & a & a & a \\ L_2 & a & u^* & b & b & b \\ L_3 & b & a & u^* & a & b \\ L_4 & b & a & b & u^* & a \\ L_5 & b & a & a & b & u^* \end{array}$$

with the types L_3, L_4, and L_5 receiving equal probability weight and the other two also positive probability weight.

A.14 There is no ESS when $|\Theta| = 6$ in the conflict case

Suppose the base game is one of conflict with $\frac{a+b}{2} > d > b$ and $|\Theta| = 6$. We now show that this meta game has no ESS. By Lemma 3 any ESS must have full support on all six types. We then show that any candidate ESS that satisfies properties a and b of Lemma 3 either has a dominated type or has type game equilibrium that does not have full support. In either case it then follows that the type game cannot have a full support ESS. This is immediate in the dominated type case and true in the other case by the fact that a full support ESS is necessarily the unique Nash equilibrium of a game (see e.g. Weibull (1995, Proposition 2.2)).

There are a series of cases to go through. First, consider the case that there is one type who plays H against all other types. Then this type dominates all other types, and we arrive at a contradiction. Second, suppose that there is one type who plays D against all other types. Then this type is dominated by all other types, a contradiction.

Third, consider the case that one type plays H against all but one other type. Then, if we want to avoid having dominated types, the type game must have the following substructure.

	L_1	L_2	L_3	L_4	L_5	L_6
L_1	u^*	b	a	a	a	a
L_2	a	u^*	b	b	b	b
L_3	b	a	u^*			
L_4	b	a		u^*		
L_5	b	a			u^*	
L_6	b	a				u^*

Note that types the four types 3 to 6 are all treated equally by types 1 and 2. They can only differ in how they play against each other. The problem, thus, reduces to considering these four types only and by the argument above there is no type structure with four types in which there is no dominated type.

Fourth, a similar argument can be made when we consider the case that one type plays D against all but one other type. This also leads to the existence of a dominated type in much the same way as in the previous case.

The remaining cases must then all have that every type plays H against at least two and at most three opponents. Given that the total number of H plays in the matrix must be 15 we must have exactly three types who play H against two opponent types and exactly three types who play H against three opponent types. Let us call the first group the $2H$-group and the latter the $3H$-group. There are now, without loss of generality, four cases. Each group (of three types each) amongst themselves can only be either egalitarian or hierarchical. Each case leads to a different type structure, all are approximate egalitarian.

We omit reproducing the four possible type games here. We only describe the results. If both groups are hierarchical amongst themselves, this leads to the approximate egalitarian type structure in which types are allocated on a circle in a specific way (as used in the proof of Proposition 4). This type game has a dominated type. If the $3H$-group is hierarchical and the $2H$-group is egalitarian then any induced type structure has the non-egalitarian Nash equilibrium with five types in its support as described above. If the $3H$-group is egalitarian and the $2H$-group is hierarchical then any induced type structure has the egalitarian Nash equilibrium with five types in its support. If both groups are egalitarian then any induced type structure has the egalitarian Nash equilibrium with three types in its support.

A.15 Proof of Lemma 8

Proof of Lemma 8: Note that the expected payoff of any label θ_i, $i \in \{1, \ldots, |\Theta|\}$, in some group of labels Θ_j, $j \in \{1, \ldots, M\}$, can be decomposed in the probability of playing against a label in its own group Θ_j times the conditional expected payoff w_j in that case, and the probability of playing against any label not in the group and the conditional expectation in that case.

Consider a full support strategy σ of the type game induced by a pre-stable structure.

Proof of "only if" statement: Suppose there is a group Θ_j which is not within equilibrated. Then there are at least two labels which earn a different expected payoff conditional on playing in that group. But since all labels outside the group play identically against both labels, this implies that they also earn different expected payoffs overall and thus the full support strategy of the full type game cannot be equilibrated. Thus being within sub-group equilibrated is a necessary condition for σ to be equilibrated. Next we show that σ can only be equilibrated if it is inter group equilibrated. Suppose not. Then pick two labels from different groups Θ_i and Θ_j. Then both labels earn different payoffs, contradicting that σ is equilibrated.

Proof of the "if" statement: Suppose the full label support strategy σ is within sub-group equilibrated and inter-group equilibrated. Then, because of within-subgroup equilibration every label $\theta_i in \Theta_j$ earns the same expected payoff as Θ_j in the inter group type game. Furthermore all Θ_j, $j \in \{1, \ldots, M\}$ earn the same expected payoff (since σ is inter-group equilibrated), all labels earn the same expected payoff and σ is equilibrated, q.e.d.

A.16 Proof of Proposition 5

(a) Since $u^* \geq b$ for conflict games the top label (who plays hawk and earns the largest possible payoff a against all other labels) is a (at least weakly) dominant strategy in the type game and would under full label support earn strictly more than any other strategy of the type game.

(b) Since $u^* \geq b$ for conflict games a bottom label (who plays dove against all other labels) is weakly dominated by all other strategies and it thus cannot be part of any full support equilibrium of the type game.

(c) The same argument as (a) now applies to the top group in the inter-group type game.

(d) The same argument as in (a) now applies to the sub-group type game $\mathfrak{G}_{\Theta_j}$ of such a sub-group Θ_j with a top label.

(e) The same argument as in (b) now applies to the sub-group type game $\mathfrak{G}_{\Theta_j}$ of such a sub-group Θ_j with a bottom label.

A.17 Proof of Lemma 9

The average payoff in any type game induced by a pre-stable structure is given by

$$\begin{aligned}
\sum_{\theta,\theta'\in\Theta} \sigma(\theta)T_{\theta,\theta'}\sigma(\theta') &= u^* \sum_{\theta\in\Theta} (\sigma(\theta))^2 + \frac{a+b}{2} \sum_{\theta\neq\theta'} \sigma(\theta)\sigma(\theta') \\
&= u^* \left(\sum_{\theta\in\Theta} (\sigma(\theta))^2 \right) + \frac{a+b}{2} \left(1 - \left(\sum_{\theta\in\Theta} (\sigma(\theta))^2 \right) \right). \qquad (26)
\end{aligned}$$

Note that $\left(\sum_{\theta\in\Theta} (\sigma(\theta))^2\right) \in [\frac{1}{|\Theta|^2}, 1]$, under the constraint $\sum_{\theta\in\Theta} \sigma(\theta) = 1$, is minimized by σ with $\sigma(\theta) = \frac{1}{|\Theta|}$ for all $\theta \in \Theta$ and is maximized by a σ with $\sigma(\theta_T) = 1$ for one label $\theta_T \in \Theta$ and with $\sigma(\theta) = 0$ for all remaining labels $\theta \neq \theta_T$. Thus, the average payoff is a weighted average of u^* and $\frac{a+b}{2}$ and is maximized by putting as much weight as possible on the higher number of the two, q.e.d.

A.18 Proof of Proposition 6

We know from Lemma 3 and Lemma 4 that for $d > \frac{a+b}{2}$ (i.e. for cases (a) and (b) no ESS and no NSS with full label support can exist for $|\Theta| \geq 2$. Now if any NSS with more than two labels in its support would exist, then, by Lemma 5 it would also be an NSS in the game restricted to the set of labels in the support Θ_S. But in this restricted meta game it would be a full support equilibrium, a contradiction.

Part (a) and (b) follow directly from this argument.

(c) Follows directly from Proposition 2, Proposition 3, and Lemma 9.

(d) Follows directly from Proposition 2, Proposition 3, Proposition 4, and Lemma 9.

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